

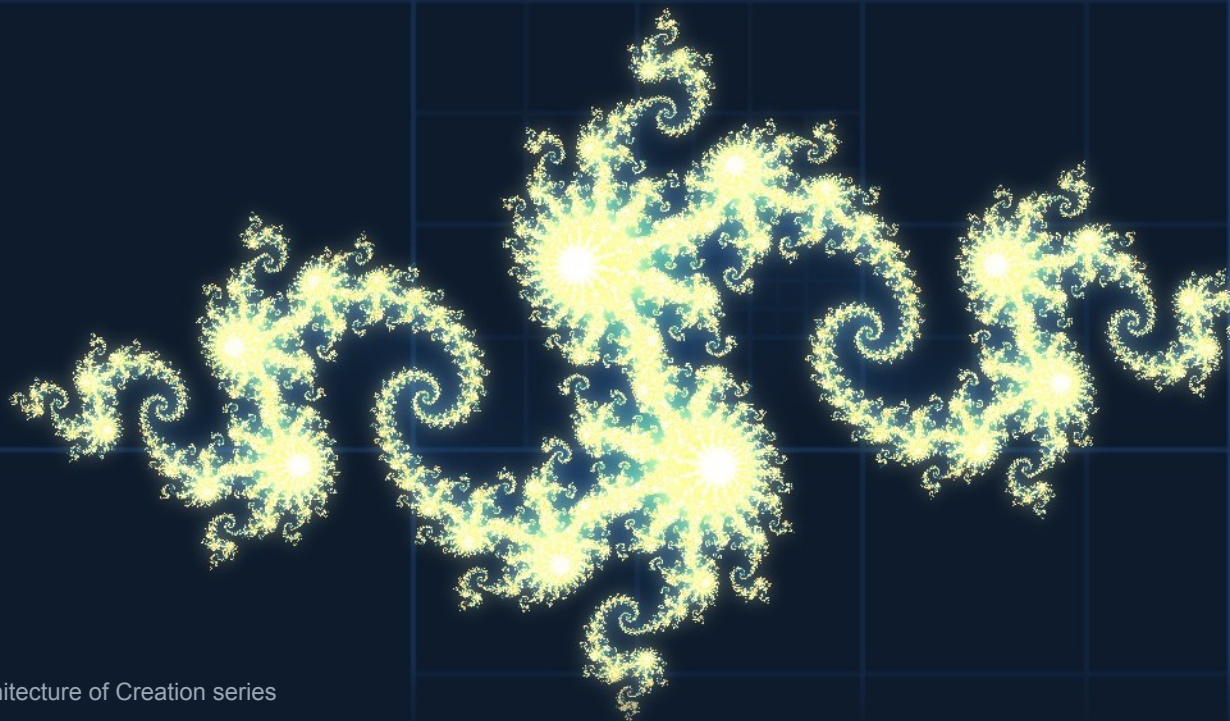
H HEURISTICS

RESEARCH REPORT · OCTOBER 2026

The Architecture of Fractals

Mathematics, Emergence, Scale and the Structure of Reality

Self-similarity · Dimension · Branching · Aggregation · Criticality · Scale · Limits



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H Heuristics · Architecture of Creation series

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About This Report

This report asks whether fractal geometry provides a unifying way to understand how patterns emerge, repeat and organize across the scales of reality, from the branching of blood vessels and rivers to the networks of cities and the large-scale distribution of matter in the universe. It belongs to the H Heuristics [Architecture of Creation](#) series. The foundational report argued that reality produces recurring forms because matter, energy, information and development operate within a constrained set of mathematical relationships and physical laws: constraints define a space of viable forms, generative processes explore that space, and filters decide what persists. Later reports applied that model to the cosmos in [The Architecture of Creation in the Universe](#), to biology in [The Architecture of Life](#), to a coupled planet in [The Architecture of Earth](#), to thermodynamics in [The Architecture of Entropy](#) and to human societies in [The Architecture of Civilization](#). [The Mathematical Architecture of Reality](#) asked why mathematics describes the world at all, and [A Unified Architecture of Reality](#) compared every domain.

Fractals appeared in almost all of those reports: in river networks and coastlines, in the branching of lungs and trees, in the clustering of galaxies, in the size distribution of cities. None of them examined fractals in their own right. This report does. Fractals are particularly relevant to the series because they raise a striking possibility: vastly different structures can share an organizational pattern even when the processes that build them are different. If that is true, fractal geometry might be one of the clearest windows onto a general architecture of creation. If it is false, or only partly true, the series needs to know where the resemblance stops.

The objective is therefore not to claim that everything in reality is fractal. It is to investigate where fractal structure appears, what generates it, why it persists, and where the apparent similarity between systems breaks down. The report is written for researchers, students and general readers. It assumes no mathematics beyond logarithms, and every formula is explained in words.

How to read this report

Each section starts on a new page and can be read on its own. The report has four parts (Figure 1). Sections 1 to 4 set out the question, review the history of fractal ideas, explain the mathematics, and build a framework: seven generative mechanisms, a five-rung ladder for grading fractal claims, and five tests that any claim should pass. Sections 5 to 8 survey four domains where fractal structure is reported: life, Earth, the universe and civilization. Sections 9 to 11 ask the explanatory questions: when similar forms share a cause, why fractal structure persists, and where fractality breaks down. Sections 12 to 15 test the report's central hypothesis with ten original calculations, state ten testable propositions, draw out implications and give a verdict.

FOUNDATIONS

**1. Introduction**

The question, the hypothesis and what is not claimed

Section 1

**2. Literature Review**

From mathematical monsters to a geometry of nature

Section 2

**3. Mathematical Foundations**

Self-similarity, dimension, iteration and multifractals

Section 3

**4. The Theoretical Framework**

Generative mechanisms, a ladder of claims and five tests

Section 4

WHERE FRACTALS APPEAR

**5. Life and Biological Organization**

Vessels, lungs, plants, neurons and genomes

Section 5

**6. Earth and Natural Systems**

Coastlines, rivers, clouds, turbulence and earthquakes

Section 6

**7. The Universe**

Filaments, clusters, voids and the scale of homogeneity

Section 7

**8. Civilization and Networks**

Cities, streets, markets and scale-free networks

Section 8

EXPLANATION

**9. Similar Forms, Different Causes**

Resemblance, mechanism and universality

Section 9

**10. Why Fractal Structure Persists**

Function, optimization, selection and criticality

Section 10

**11. Where Fractality Breaks Down**

Cutoffs, scaling ranges and statistical traps

Section 11

TESTS AND VERDICT

**12. Testing the Hypothesis**

Ten computational tests and a scorecard

Section 12

**13. Hypotheses and Research Agenda**

Ten testable propositions

Section 13

**14. Implications**

For science, design, medicine and the series

Section 14

**15. Conclusion**

An architecture, not a universal law

Section 15

Figure 1. The structure of the report. Four foundational sections establish the question and the framework, four survey the domains where fractal structure appears, three explain it and bound it, and four test the hypothesis and draw conclusions.

Source: *H Heuristics*.

Blue underlined text is a hyperlink. Where a specific result is cited, links go to the DOI record of the primary literature. Where a concept is introduced, links go to a reference article, usually Wikipedia for mathematical and physical concepts. Every DOI was checked against its registered title before publication, and a full reference list follows Section 15.

Evidence grades

Claims are graded on the same four-level scale used throughout the series.

Table 1. Evidence grades used in this report

Grade	Meaning	Example from this report
A: Established	Proven mathematically, or confirmed repeatedly by independent measurements over a wide range of scales.	The Koch curve has dimension $\log 4 / \log 3$; the universe is statistically homogeneous above about 100 megaparsecs
B: Well supported	Strong evidence from several lines of work, with unresolved detail or some dissent.	River networks obey Hack's and Horton's scaling laws over several orders of magnitude
C: Plausible or contested	A credible argument and partial evidence; specialists actively disagree.	Animal metabolic rate scales as mass to the $3/4$ power because circulatory networks are fractal
D: Speculative	A hypothesis or interpretation offered for evaluation, labeled as such wherever it appears.	Fractal organization is a deep architecture common to all of reality

The grade describes the state of knowledge, not the importance of a claim. A system can be graded B for 'fractal-like over a limited range' and D for 'fractal at every scale' at the same time; the report grades the specific claim being made.

A note on the figures and calculations

The report contains 28 figures and 18 tables. Twenty figures present calculations made for this report, written in Python. They construct the classic fractals and measure their dimensions by box counting; measure Richardson's coastline effect on random curves of known dimension; carve drainage networks into a fractal landscape and test Hack's law; cut clouds from a fractal surface and measure their area–perimeter relation; run a self-organizing sandpile; grow diffusion-limited aggregates and critical percolation clusters; compare two objects with the same dimension and different origins; evolve a cosmic web; build a hierarchical galaxy model that becomes homogeneous at large scales; grow preferential-attachment networks and Zipf-distributed city systems; test whether a lognormal can pass as a power law; show that non-fractal objects can yield convincing 'fractal dimensions'; compute the scaling exponents of finite branching networks; and measure the material saved by hierarchical supply networks. Each caption states what was computed and with what assumptions, and every number quoted from the models can be regenerated. The other figures are diagrams drawn for this report.

How to cite

Hughes, H. (2026). *The Architecture of Fractals: Mathematics, Emergence, Scale and the Structure of Reality*. H Heuristics Research Report, Architecture of Creation series. The report synthesizes published research and presents original calculations and propositions, which are labeled as such.

Executive Summary

Abstract

A fractal is a shape whose detail persists as it is magnified. Mathematicians built the first examples in the late nineteenth century as curiosities: curves with no tangents, sets with no length, a snowflake with an infinite perimeter enclosing a finite area. In 1967 Benoit Mandelbrot argued that these 'monsters' were in fact the typical geometry of nature, and the idea spread rapidly. Fractal structure has since been reported in coastlines, rivers, clouds, turbulence, lungs, blood vessels, neurons, plants, genomes, galaxies, cities, markets and networks. This report asks whether that recurring geometry reveals a unifying architecture. It distinguishes three levels of claim: mathematical (established), physical and biological (well supported, within limits) and foundational (open). It identifies seven generative mechanisms that produce fractal structure, proposes a five-rung ladder for grading fractal claims, and tests a Fractal Architecture Hypothesis with ten original calculations. The report concludes that fractal geometry is a powerful common language and a genuine but bounded architecture. Similar forms arise because a small number of processes, above all branching for transport, aggregation, criticality, cascades and proportional growth, recur across domains. Where systems share a mechanism class they share exponents, and the resemblance is deep. Where they merely share a look, the resemblance is shallow. Real fractals always end at an upper and a lower scale, and reality as a whole is not fractal.

The central question

Central question. Does fractal geometry provide a unifying way to understand how patterns emerge, repeat and organize across different scales of reality, from branching structures in nature to networks, landscapes, biological systems and the large-scale distribution of matter in the universe?

The question has three parts. *Where* does fractal structure genuinely appear, as opposed to being claimed on weak evidence? *What* generates it, and are the generators the same across domains? And *why* does it persist: is it selected for function, an inevitable by-product of growth, or a signature of systems poised at a critical point? A unifying account would need positive answers to all three.

The Fractal Architecture Hypothesis

Central hypothesis. Across multiple domains of reality, recurring structures can emerge across different scales through processes such as branching, recursive growth, aggregation, feedback and constrained optimization. Fractal geometry provides a mathematical language for identifying and comparing some of these structures. Where different systems share a generative mechanism, they will share measurable scaling properties; where they do not, their resemblance will fail quantitative tests.

The hypothesis is deliberately testable. It does not say that everything is fractal. It predicts that fractal similarity between systems will *track* similarity of mechanism, and that it will fail in specific, detectable ways when mechanisms differ. Section 12 tests it.

Principal findings

1. **Fractals are mathematically exact but physically approximate (Grade A).** Ideal fractals have precisely defined non-integer dimensions: the Koch curve 1.262, the Sierpiński triangle 1.585. Every physical fractal is a finite approximation that holds between an upper and a lower cutoff. Box counting on the report's constructions recovered known dimensions to within about 3 per cent, but a finite random walk that should have dimension 2 measured only 1.735 (Figure 7).
2. **Fractal structure is real and widespread within limits (Grade B).** Cloud and rain areas follow a fractal area–perimeter law over six orders of magnitude of area. River networks obey Hack's and Horton's laws. Bronchial and arterial trees branch in near-self-similar fashion for 15 to 25 generations. Galaxy clustering follows a power law from roughly a tenth of a megaparsec to about ten megaparsecs. These are robust, repeatedly measured regularities.
3. **A small number of mechanisms generates most of it (Grade B).** Seven processes account for nearly every well-documented case: branching for transport, recursive growth, diffusion-limited aggregation, criticality, cascades, erosion and competing forces, and proportional growth (Figure 9). The report's models show that each can produce fractal statistics from simple rules: drainage carved into a fractal landscape reproduced Hack's exponent (0.558 against 0.5–0.6 in real basins), and a sandpile organized itself to produce avalanches spanning four orders of magnitude with no tuning.
4. **Shared mechanism means shared numbers; shared looks do not (Grade A–B).** Diffusion-limited aggregation produces dimension near 1.7 whether the material is zinc, an electrical discharge or air pushed into oil, because all three are governed by the same equation. The report's aggregate measured 1.733. This is universality, and it is the strongest form of fractal unity. By contrast, an aggregate and a recursively cut 'dust' built to the same dimension (1.64 and 1.644) differ in texture by a factor of about two (Figure 22). A single dimension cannot identify a mechanism.
5. **Function explains persistence in living systems (Grade B).** Branching hierarchies supply a volume with far less material than direct connections: in the report's calculation, a hierarchy reached a million points with 261 times less pipe than a star of separate pipes (Figure 24). Murray's law of minimum work, $r^3 \propto Q$, follows from balancing pumping cost against upkeep. Fractal-like structure persists in organisms because it is cheap, robust and space-filling, and because development can generate it by repeating one rule.
6. **The famous metabolic 3/4 law is contested (Grade C).** The West–Brown–Enquist model derives a 3/4 exponent from space-filling, area-preserving networks. But real data show curvature and taxon-dependent exponents, and the report's calculation shows that the 3/4 value is a limit approached only by infinitely large networks: model networks with realistic numbers of levels gave local exponents between 0.739 and 0.806 (Figure 12).
7. **The universe is fractal only over a limited range (Grade A).** Galaxy clustering has a correlation dimension well below 3 on scales of a few megaparsecs, about 1.2 where clustering is strongest, rising with scale. Large surveys find a transition to homogeneity, dimension 3, above roughly 100 megaparsecs (about $70 h^{-1}$ Mpc). The report's hierarchical model reproduces such a transition (Figure 19). Claims that the cosmos is fractal at all scales are not supported.
8. **Many fractal claims are statistically weak (Grade A–B).** A survey of published experimental dimensions found typical scaling ranges of little more than one decade. The report shows that random discs, which are not fractal, give a 'dimension' of 1.5 with a near-perfect straight-line fit over 1.3 decades (Figure 25), and that a lognormal distribution can mimic a power law until a large sample and a likelihood test separate them (Figure 26). Claims of scale-free networks, fractal cities and fractal art have all weakened under such tests.

- 9. Reality is better described as multiscale than as fractal (Grade B).** Many natural systems are multifractal, needing a spectrum of dimensions, or self-affine, scaling differently in different directions, or fractal only between characteristic scales set by physics. The most accurate general statement is that reality has recurring scale-dependent structure, of which strict fractals are one idealized case.

The verdict

Fractal geometry does provide a unifying language, and in one important sense a unifying architecture: a handful of generative mechanisms recur across life, Earth, the cosmos and society, and wherever the same mechanism operates the same scaling numbers appear. That is a real discovery about how reality builds structure. But the unity is of processes, not of shapes. Fractal resemblance is evidence of a shared principle only when it survives quantitative tests: matching exponents over two or more decades, a mechanism that reproduces them, and agreement at the cutoffs. Much of the popular case for a fractal universe fails those tests. The report therefore grades the physical and biological level of the hypothesis B and the foundational level D: plausible, productive and unproven.

In one sentence. Fractals reveal not that everything is the same shape, but that a few ways of building things recur everywhere, and fractal geometry is how those recurring ways can be recognized and measured.

01

INTRODUCTION



1. Introduction

1.1 A coastline with no length

In the 1950s the English physicist Lewis Fry Richardson was trying to find out whether the probability of war between neighbouring countries depended on the length of their shared border. He ran into an unexpected problem. Published border lengths disagreed wildly: Spain and Portugal reported their common frontier as 987 and 1,214 kilometres. The discrepancy was not carelessness. Richardson found that the measured length of a coast or frontier grew steadily as the measuring stick shrank, following a power law. A rough coast has no well-defined length at all; it has a length *at a given resolution*. Richardson's data were published posthumously in 1961 in an obscure yearbook. In 1967 Benoit Mandelbrot used them in a short paper in *Science* with a provocative title, [How Long Is the Coast of Britain?](#), and showed that the exponent of Richardson's power law could be read as a *dimension* lying between 1 and 2. For the west coast of Britain it was about 1.25.

That observation launched a field. Mandelbrot coined the word *fractal* in 1975, from the Latin *fractus*, broken, and in *The Fractal Geometry of Nature* (1982) argued that 'clouds are not spheres, mountains are not cones, coastlines are not circles'. The roughness of nature, he proposed, is not noise around ideal Euclidean shapes. It is a geometry in its own right, with its own measures and its own laws. Within two decades fractal structure had been reported in almost every science.

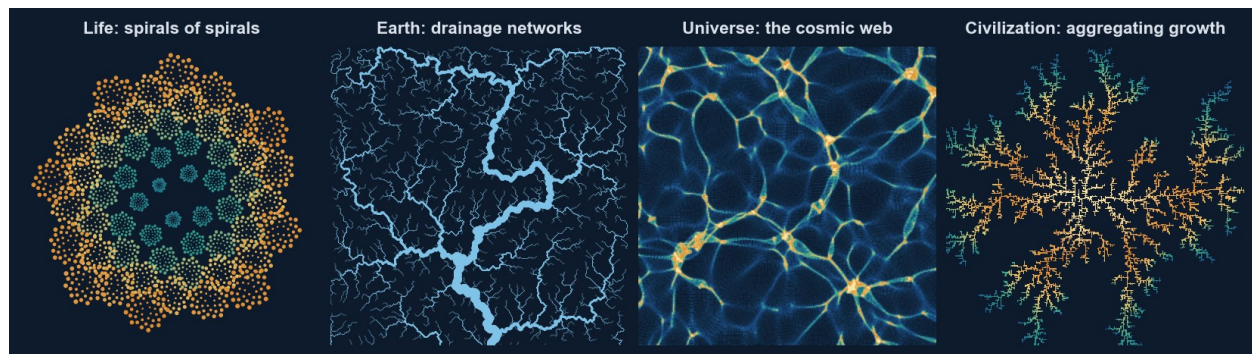


Figure 2. Four generated structures that recall four domains of the report. From left: a spiral of spirals like a Romanesco inflorescence; a drainage network carved into a fractal landscape; a cosmic web of filaments and voids; an aggregate grown by random walkers, resembling an urban footprint. Each was produced by a different rule.

Source: H Heuristics calculations. Plant: nested golden-angle spirals. River: priority-flood drainage on a fractional Brownian surface (Section 6). Cosmic web: Zel'dovich approximation (Section 7). Aggregate: diffusion-limited aggregation, 12,000 particles (Section 9).

1.2 The central question

Figure 2 shows four images that look as if they belong together. Each has branching or filamentary structure, detail at many scales and an irregularity that a ruler and compass cannot describe. Yet each

was produced by a different process: a developmental rule, erosion, gravity and random diffusion. That is the puzzle this report investigates.

Central question. Does fractal geometry provide a unifying way to understand how patterns emerge, repeat and organize across different scales of reality?

The question matters to the Architecture of Creation framework because the framework claims that recurring forms are evidence of recurring constraints and processes. Fractals are the most dramatic case of recurrence. If the similarity between a lung, a river and the cosmic web reflects a shared principle, fractal geometry would be a key to the architecture. If the similarity is coincidental, or produced by the human eye's appetite for pattern, fractals are a warning about how easily resemblance is mistaken for explanation.

1.3 The Fractal Architecture Hypothesis

The report evaluates the following hypothesis.

The Fractal Architecture Hypothesis. Across multiple domains of reality, recurring structures can emerge across different scales through processes such as branching, recursive growth, aggregation, feedback and constrained optimization. Fractal geometry provides a mathematical language for identifying and comparing some of these structures.

The hypothesis has three distinct levels (Figure 3), and much confusion in the literature comes from sliding between them.

- **Mathematical.** Fractals can exhibit self-similarity, recursive structure and non-integer dimension. This is a matter of proof and is not in doubt.
- **Physical and biological.** Particular processes generate fractal-like structures, including river networks, branching organisms, turbulent flows and porous or aggregated materials. This is an empirical claim, true in many cases and always within a limited range of scales.
- **Foundational.** Fractal organization may reveal a deeper, general architecture of reality. This is an open research question, not a finding.

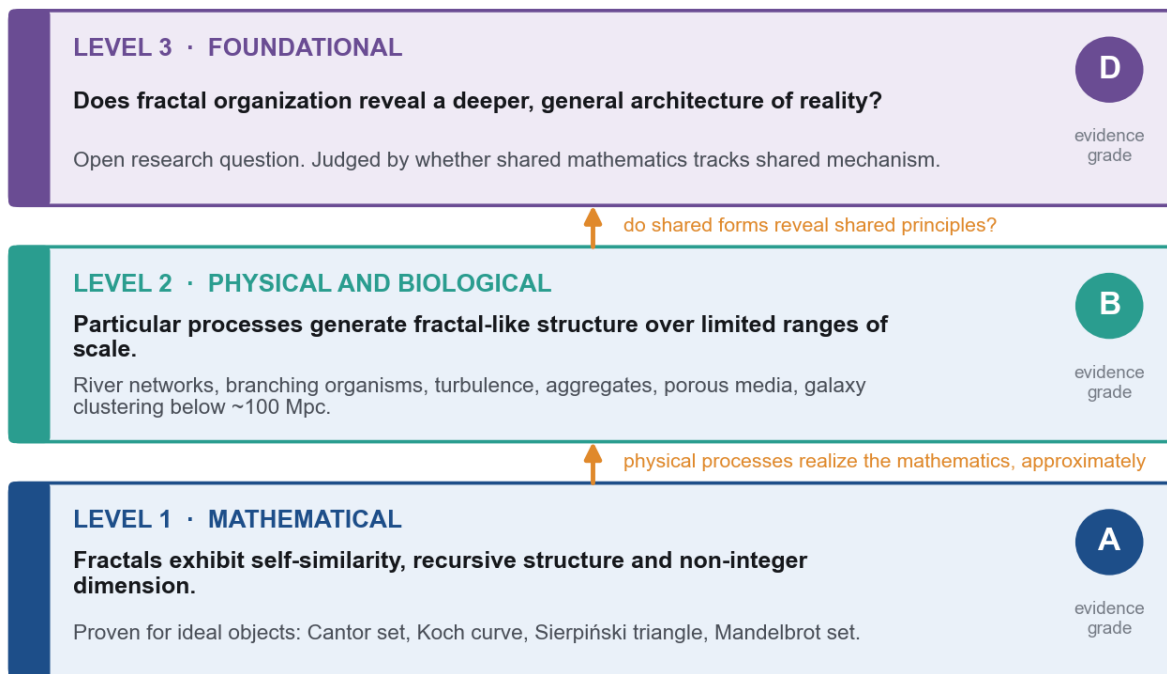


Figure 3. Three levels of the Fractal Architecture Hypothesis. The mathematical level is proven; the physical and biological level is well supported within limited ranges of scale; the foundational level is open. Arguments often move from the first to the third without passing through the second.

Source: *H Heuristics*.

The distinction matters for a simple reason. A pattern that *looks* fractal is not necessarily a mathematically established fractal, and similar shapes do not necessarily share a common cause. The report's method is to keep the three levels apart and to ask, at every step, what evidence would move a claim from one level to the next.

1.4 What this report does not claim

Table 2. What the report does and does not claim

The report argues	The report does not argue
Fractal geometry is an exact branch of mathematics with well-defined measures of dimension	That any physical object is a true fractal at all scales
Many natural and social systems are fractal-like over a limited, measurable range of scales	That everything in nature, or the universe as a whole, is fractal
A small number of generative mechanisms produces most documented fractal structure	That all fractal-looking systems share one mechanism
Shared exponents within a universality class reveal a genuinely shared principle	That shared appearance or a shared dimension proves shared cause
Fractal structure persists in living systems largely because of its function	That evolution or design 'aims' at fractal form
Fractals are a useful language for the Architecture of Creation framework	That fractals are the foundation of reality

The left column states positions the report defends with graded evidence. The right column lists claims that circulate widely but that the evidence reviewed here does not support.

1.5 Method and scope

The report combines four methods. First, a synthesis of primary literature across mathematics, physics, geomorphology, physiology, ecology, cosmology, urban science and network science. Second, a framework of generative mechanisms and a graded ladder of claims (Section 4). Third, ten original calculations that construct fractals, measure them and deliberately try to break the hypothesis (Section 12). Fourth, ten testable propositions for future research (Section 13). The scope is deliberately broad, because the question is about whether a pattern unifies domains, and that cannot be judged one domain at a time. The cost of breadth is depth: each domain section is a survey, not a review, and points to the specialist literature by hyperlink.

02

LITERATURE REVIEW



2. Literature Review

2.1 Mathematical monsters, 1872–1918

Fractal ideas began as counterexamples. In 1872 Karl Weierstrass presented a [function that is continuous everywhere but differentiable nowhere](#): its graph has no tangent at any point, however closely one looks. In 1883 Georg Cantor constructed a set by repeatedly removing the middle third of every interval, leaving a 'dust' of points with zero total length that is nonetheless uncountably infinite ([Cantor 1883](#)). In 1904 Helge von Koch built a continuous curve with no tangents by repeatedly replacing each straight segment with four shorter ones, and in 1915 Waław Sierpiński described his triangle of triangles. Many mathematicians regarded such objects as pathological, and they became known as 'monsters'.

The tool that would later measure them arrived in 1918, when Felix Hausdorff defined a notion of dimension that need not be an integer ([Hausdorff 1918](#)). In the same year Gaston Julia, and soon after Pierre Fatou, studied what happens when a simple function of a complex number is applied over and over again, producing the sets now named after Julia. Without computers to draw them, their extraordinary structure went largely unseen for sixty years.

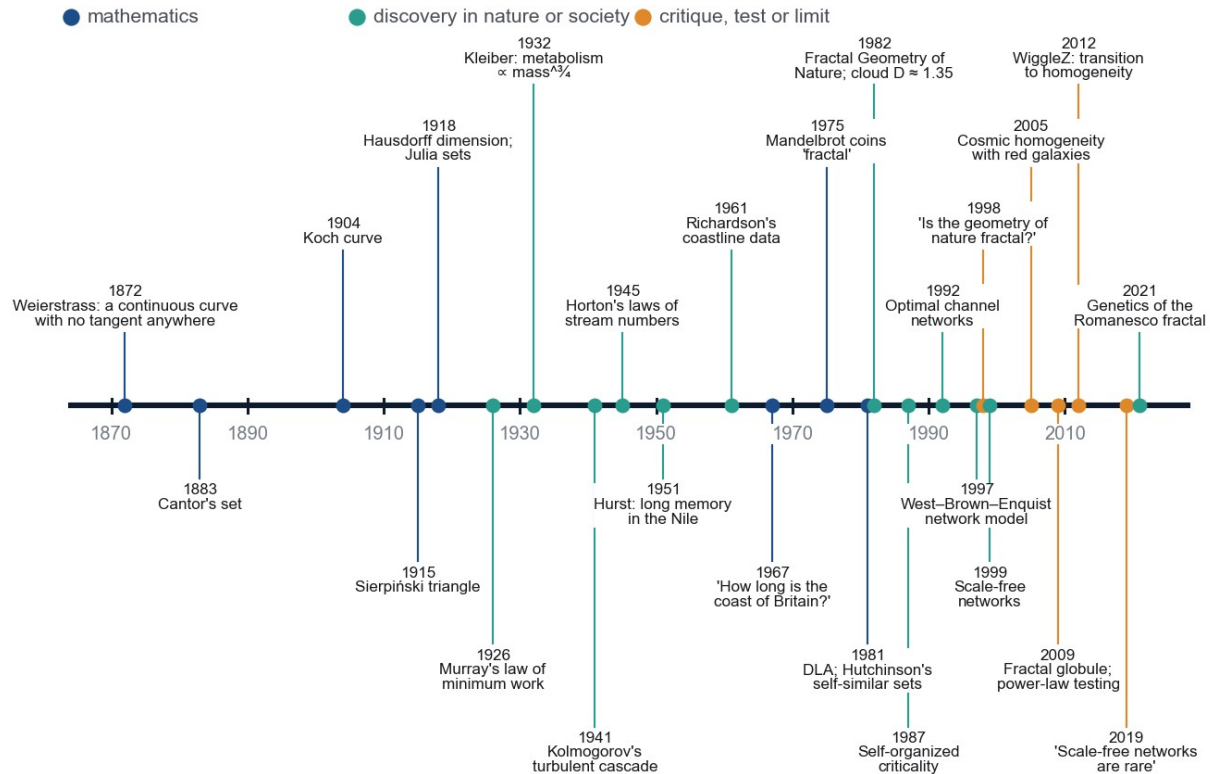


Figure 4. A timeline of fractal ideas. Mathematical constructions came first; discoveries of fractal-like scaling in nature and society accumulated through the twentieth century; critiques and tests that bound the claims have dominated since the late 1990s.

Source: H Heuristics, from the primary sources cited in Section 2.

2.2 Scaling laws in nature before fractals

Long before the word fractal existed, scientists were finding power laws that would later be read as fractal signatures. In 1926 Cecil Murray showed that blood vessels minimize the combined cost of pumping blood and maintaining it if the cube of a parent vessel's radius equals the sum of the cubes of its daughters' radii ([Murray 1926](#)). In the same year Richardson found that the effective diffusivity of turbulent air grows with the separation of two particles to the 4/3 power ([Richardson 1926](#)), and in 1941 Andrei Kolmogorov explained the energy spectrum of turbulence as a cascade of eddies within eddies ([Kolmogorov 1941](#), [English translation 1991](#)). In 1932 Max Kleiber found that animal metabolic rate scales with body mass to roughly the 3/4 power ([Kleiber 1932](#)). In 1945 Robert Horton showed that the numbers and lengths of stream segments in a river basin fall in geometric ratios as one moves up the branching hierarchy ([Horton 1945](#)); in 1957 John Hack found that the length of a river's main stream grows with its drainage area to about the 0.6 power ([Hack 1957](#)). In 1951 the hydrologist Harold Hurst discovered long-range memory in the annual floods of the Nile ([Hurst 1951](#)).

These results were found independently, in different disciplines, with no common language. What fractal geometry later supplied was the recognition that each was a statement about how something scales, and that scaling without a characteristic length is the signature of self-similarity.

2.3 Mandelbrot and the geometry of nature, 1963–1982

Mandelbrot's genius lay in connecting these threads. His 1963 analysis of cotton prices showed that price changes have far heavier tails than the normal distribution assumed by economists, and that their statistics look similar over days, months and years ([Mandelbrot 1963](#)). With John Van Ness he formalized [fractional Brownian motion](#), a family of random processes whose roughness is set by a single exponent, the Hurst exponent H . The coastline paper of 1967 introduced statistical self-similarity to a wide audience. In 1974 he proposed that turbulent dissipation is concentrated on a fractal set ([Mandelbrot 1974](#)), and in 1980 he published the first images of the set that bears his name ([Mandelbrot 1980](#)). *The Fractal Geometry of Nature* gathered the program into a manifesto.

Two mathematical developments gave the program rigor. John Hutchinson showed in 1981 that any finite set of contracting maps defines a unique self-similar set ([Hutchinson 1981](#)), and Michael Barnsley showed how such [iterated function systems](#) could generate natural-looking shapes such as ferns from a handful of numbers. In 1983 Peter Grassberger and Itamar Procaccia gave a practical way to measure the dimension of a set of points, the [correlation dimension](#), which became the standard tool for strange attractors and later for galaxy surveys ([Grassberger and Procaccia 1983](#)).

2.4 Physics of fractal growth, 1979–1987

A second wave asked not whether nature is fractal but *how* fractals form. In 1979 Forrest and Witten found that smoke particles aggregate into clusters with a non-integer dimension ([Forrest and Witten 1979](#)). In 1981 Thomas Witten and Leonard Sander proposed [diffusion-limited aggregation](#), or DLA: particles wander randomly and stick when they touch a growing cluster. The model produced tenuous, branching clusters with a dimension close to 1.7 in two dimensions, and the same structure soon turned up in [dielectric breakdown](#), [zinc electrodeposition](#) and [viscous fingering in porous media](#). Sandstone pore spaces were found to be fractal ([Katz and Thompson 1985](#)), as were the surfaces of many catalysts and adsorbents ([Pfeifer and Avnir 1983](#)).

Mathematicians and physicists also realized that a single dimension is often not enough. Hentschel and Procaccia introduced an [infinite family of generalized dimensions](#), and Halsey and colleagues showed how to describe a [multifractal measure](#) by its spectrum of singularities. Meneveau and Sreenivasan applied the idea to turbulence with a simple [multiplicative cascade](#) ([Stanley and Meakin 1988](#); [Sreenivasan 1991](#)). In 1987 Per Bak, Chao Tang and Kurt Wiesenfeld proposed [self-organized criticality](#): driven systems with thresholds, like a sandpile, organize themselves to a critical state where events of all sizes occur, offering a general explanation for fractal structure and power laws in nature.

2.5 Fractal biology and the metabolic network debate

In biology, Ewald Weibel and Domingo Gomez had measured the [human airway tree](#) generation by generation in 1962; West, Bhargava and Goldberger reinterpreted it as a fractal ([West et al. 1986](#), and Weibel later argued that fractal geometry is a [design principle of living organisms](#). Ary Goldberger and colleagues found fractal, long-memory fluctuations in healthy heartbeats that are lost in disease and ageing ([Goldberger et al. 2002](#); [Ivanov et al. 1999](#)). Aristid Lindenmayer's [L-systems](#) showed that recursive rewriting rules can grow realistic plants.

The most ambitious biological claim came in 1997, when Geoffrey West, James Brown and Brian Enquist derived Kleiber's 3/4 law from the geometry of space-filling, hierarchically branching networks ([West et al. 1997](#); [for plants, 1999](#)). The model became the foundation of a [metabolic theory of ecology](#). It has also

been vigorously contested: re-analyses found exponents nearer $2/3$ for mammals ([Dodds et al. 2001](#); [White and Seymour 2003](#)), [curvature in the log-log relation](#), and [finite-size corrections](#) that move the model's own prediction away from $3/4$. Section 5 returns to the debate.

2.6 Networks, cities and the cosmos

Fractal and scaling ideas spread to human systems in the 1980s and 1990s. Michael Batty and Paul Longley measured the [fractal boundaries of cities](#), and Makse, Havlin and Stanley [modelled urban growth](#) as correlated percolation. Xavier Gabaix explained [Zipf's law for city sizes](#) as a consequence of random proportional growth. In 1999 Albert-László Barabási and Réka Albert showed that growth with preferential attachment produces [scale-free networks](#), and the Faloutsos brothers reported [power laws in the Internet's topology](#). Song, Havlin and Makse later found [self-similarity in complex networks](#) under renormalization.

In cosmology, the galaxy correlation function was found to be a power law ([Davis and Peebles 1983](#)), equivalent to a fractal distribution over a range of scales. Luciano Pietronero and colleagues argued that the universe might be [fractal on all observed scales](#) ([Sylos Labini et al. 1998](#)), contradicting the cosmological principle. The debate was settled by large redshift surveys, which found a transition to homogeneity ([Hogg et al. 2005](#); [Scrimgeour et al. 2012](#)).

2.7 The critical turn, 1998 to the present

Since the late 1990s much of the most important work has been critical. In 1998 David Avnir and colleagues surveyed experimental fractal dimensions reported in the *Physical Review* journals and found that most rested on scaling ranges of little more than a decade, and asked bluntly, [Is the geometry of nature fractal?](#) Halley and colleagues documented the [uses and abuses of fractal methods in ecology](#). Clauset, Shalizi and Newman showed that many published [power laws in empirical data](#) fail rigorous tests, and Stumpf and Porter set out [critical truths about power laws](#). Broido and Clauset tested nearly a thousand networks and concluded that [scale-free networks are rare](#), prompting a lively exchange ([Holme 2019](#)). The fractal analysis of Jackson Pollock's paintings ([Taylor et al. 1999](#)) was challenged on statistical grounds ([Jones-Smith and Mathur 2006](#)).

The critical turn has not overturned fractal geometry. It has sharpened it. The field now asks for wider scaling ranges, explicit cutoffs, uncertainty estimates, comparison with alternative distributions and, above all, mechanisms. That is the standard this report adopts.

2.8 What the literature leaves open

Three gaps motivate this report. First, the literature is fragmented by domain: geomorphologists, physiologists and cosmologists rarely compare their fractals. Second, there is no common scale for grading fractal claims, so a pattern seen over one decade in a photograph and a universality class confirmed in many laboratories are both called 'fractal'. Third, the question that matters most for the Architecture of Creation framework, *when does shared geometry imply shared cause?*, is usually asked rhetorically rather than tested. The theoretical framework in Section 4 addresses all three, after Section 3 sets out the mathematics.

03

MATHEMATICAL FOUNDATIONS



3. Mathematical Foundations

3.1 The question

Research question. What exactly is a fractal, how is fractal structure measured, and what do the measurements mean?

Fractals are structures whose complexity persists across changes in observational scale. Many are generated by repeating a rule; others arise from random processes or physical dynamics. Three concepts form the foundation of the rest of the report: self-similarity, fractal dimension, and recursion or iteration. A fourth, multifractality, is needed for many real systems. This section explains each in turn and then sets out the practical problems of measurement that recur throughout the report.

3.2 Self-similarity: exact, statistical and limited

A structure is [self-similar](#) if parts of it resemble the whole at different scales. The resemblance comes in three strengths (Figure 5). In *exact* self-similarity, a magnified part is an identical copy of the whole: every corner of the Sierpiński triangle is a half-size Sierpiński triangle. In *statistical* self-similarity, a magnified part is not identical but has the same statistical properties: zoom into a random walk, rescale it appropriately, and the result is indistinguishable in distribution from the original, though it is a different curve. Most natural fractals, including coastlines, clouds and topography, are of this kind. In *limited* self-similarity, the resemblance holds only between an upper scale, such as the size of the organism, and a lower scale, such as the size of a cell. All physical fractals are limited.

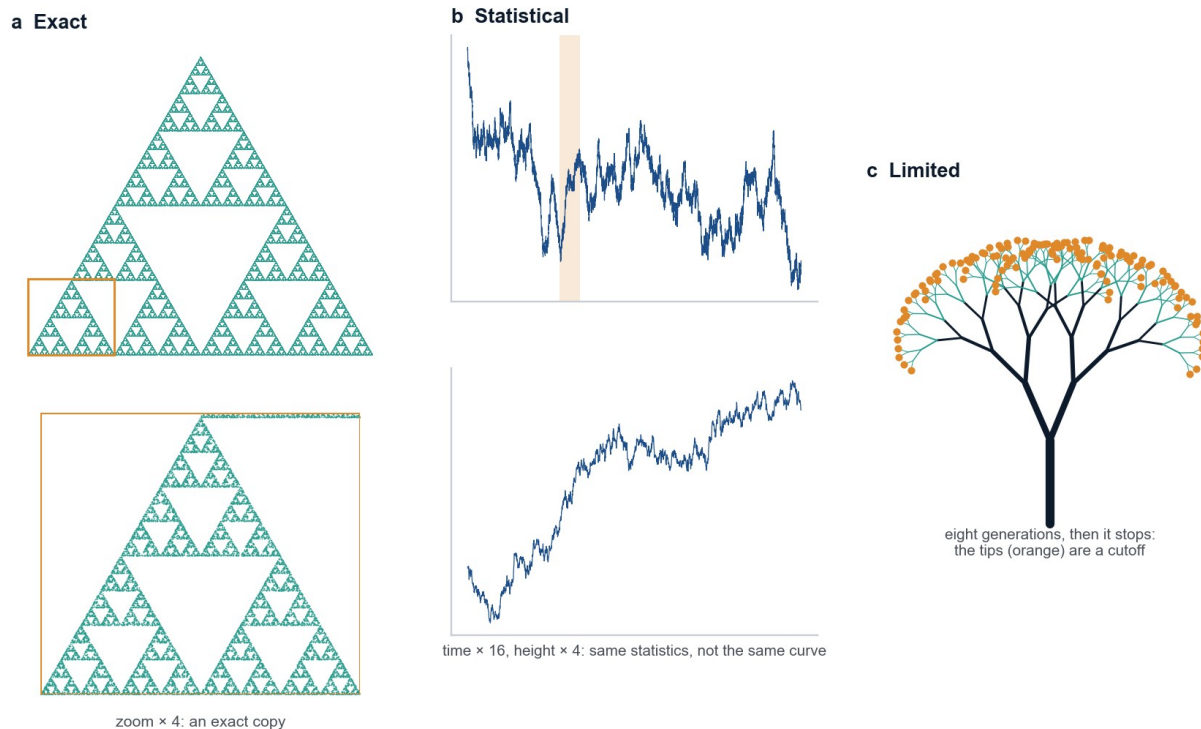


Figure 5. Three kinds of self-similarity. (a) Exact: the corner of a Sierpiński triangle, magnified four times, is an identical copy of the whole. (b) Statistical: a section of a random walk, stretched 16 times in time and 4 times in height, has the same statistics as the whole but is a different curve. (c) Limited: a branching tree repeats its rule for eight generations and stops; real fractals always have such cutoffs.

Source: H Heuristics calculations. (a) chaos-game construction, 150,000 points; (b) Gaussian random walk of 32,768 steps; (c) binary tree with branch-length ratio 0.74 and small random angle noise.

A subtle variant matters for many applications. A curve such as the graph of a random walk is *self-affine*, not self-similar: to make a magnified part look like the whole, the horizontal and vertical axes must be stretched by different factors (16 and 4 in Figure 5b). Topographic profiles, river longitudinal profiles, financial time series and many fracture surfaces are self-affine. Their dimension depends on how it is measured, a point that has caused much confusion in the empirical literature ([Mark and Aronson 1984](#); [Gneiting and Schlather 2004](#)).

3.3 Fractal dimension

The defining number of a fractal is its [fractal dimension](#), a measure of how the structure fills space as it is examined at finer scales. For ordinary shapes, dimension tells how the number of small pieces needed to cover an object grows as the pieces shrink. Halve the size of the pieces and a line needs twice as many, a square four times as many, a cube eight times as many: 2^1 , 2^2 , 2^3 . The exponent is the dimension. For an ideal self-similar fractal composed of N copies of itself, each reduced by a scale factor r , the same logic gives the *similarity dimension*:

$$D = \log N / \log(1/r) \quad (1)$$

The Sierpiński triangle consists of three copies at half scale, so $D = \log 3 / \log 2 \approx 1.585$: it is more than a line and less than a surface. The Koch curve is four copies at one-third scale, $D = \log 4 / \log 3 \approx 1.262$. The Cantor set is two copies at one-third scale, $D = \log 2 / \log 3 \approx 0.631$. Figure 6 shows six classic fractals with their dimensions.

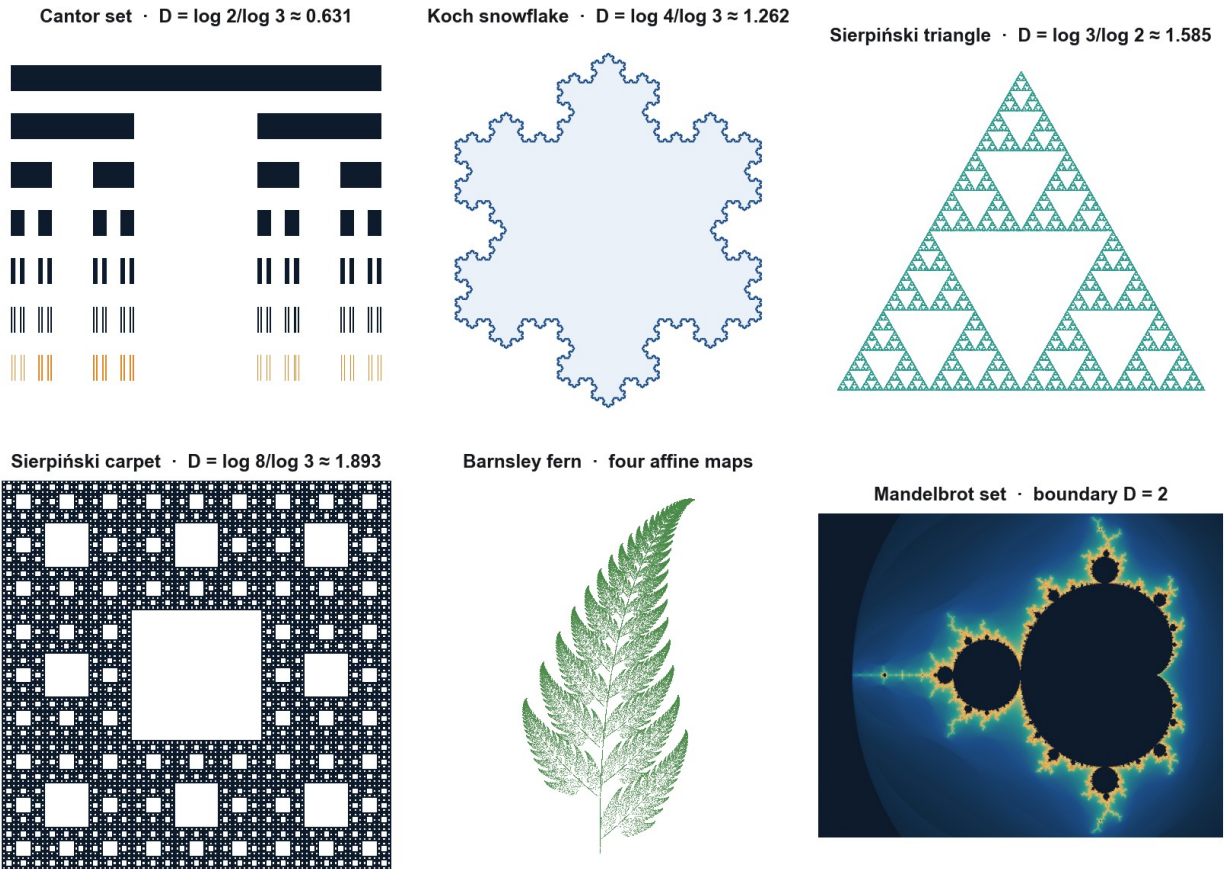


Figure 6. Six classic fractals. The Cantor set, Koch snowflake, Sierpiński triangle and Sierpiński carpet are exactly self-similar and have similarity dimensions given by Equation 1. The Barnsley fern is generated by four affine maps chosen at random. The boundary of the Mandelbrot set is so convoluted that its Hausdorff dimension is 2.

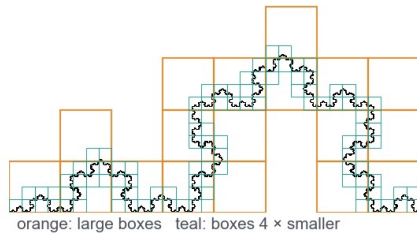
Source: H Heuristics calculations. Cantor set to 7 levels; Koch snowflake to 5 levels; Sierpiński triangle by the chaos game; carpet to 5 levels; fern by Barnsley's iterated function system; Mandelbrot set by escape time, 160 iterations.

Real objects are not built from exact copies, so their dimension must be measured. The most common method is [box counting](#): overlay grids of boxes of side ϵ , count the number $N(\epsilon)$ of boxes that contain part of the object, and repeat for many box sizes. If the object is fractal over a range of scales, $N(\epsilon)$ follows a power law there:

$$N(\epsilon) \propto \epsilon^{-D} \quad (2)$$

and D is the slope of a straight line on a plot of $\log N$ against $\log(1/\epsilon)$. Figure 7 applies the method to four objects whose dimensions are known. For the Sierpiński triangle it returned 1.582 against an exact 1.585, and for a circle 1.003 against 1. For the Koch curve it returned 1.296 against 1.262, an error of about 3 per cent caused by the mismatch between the curve's natural scale factor of 3 and the grid's factor of 2. For a random walk of four million steps, whose trail should fill the plane with dimension 2, it returned only 1.735. The trail fills the plane only in the limit of infinite length; any finite sample leaves gaps. This is a general lesson: measured dimensions of finite random objects are biased, usually low, and require care.

a Covering part of a Koch curve with boxes



b

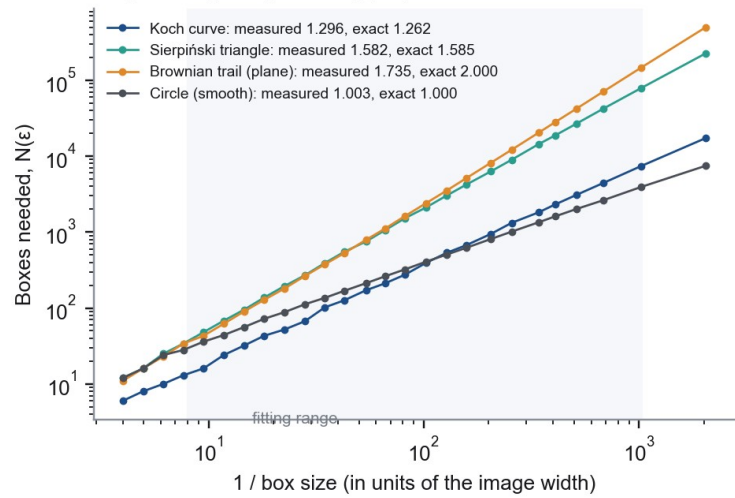
Slope of $\log N$ against $\log(1/\epsilon)$ is the dimension

Figure 7. Box counting recovers known dimensions, with caveats. (a) Boxes of two sizes covering part of a Koch curve. (b) The number of occupied boxes against inverse box size for four objects; the slope over the shaded fitting range estimates the dimension. Exact fractals are recovered to within about 3 per cent; a finite random walk falls well short of its theoretical value of 2.

Source: H Heuristics calculations on $2,048 \times 2,048$ pixel images. Koch curve to 8 levels; Sierpiński triangle 1.5 million chaos-game points; Brownian trail 4 million Gaussian steps; slopes fitted for box sizes 2–256 pixels.

Table 3. Measures of fractal dimension and where they are used

Measure	What it counts	Typical use
Similarity dimension	Number and scale factor of exact self-copies (Equation 1)	Ideal constructions: Cantor, Koch, Sierpiński
Hausdorff dimension	The critical exponent at which a covering measure jumps from infinity to zero	Rigorous mathematics; the reference definition
Box-counting dimension	Occupied grid boxes as box size shrinks (Equation 2)	Images, maps, coastlines, cities, art
Divider (ruler) dimension	Measured length as ruler length shrinks: $L(\epsilon) \propto \epsilon^{1-D}$	Coastlines and boundaries (Richardson)
Mass–radius dimension	Mass within distance r of a centre: $M(r) \propto r^D$	Aggregates, DLA, colloids, galaxy clusters
Correlation dimension D_2	Pairs of points closer than r : $C(r) \propto r^{D_2}$	Strange attractors, galaxy surveys
Area–perimeter dimension	Perimeter of many objects against their area: $P \propto A^{D/2}$	Clouds, rain areas, islands, lakes
Hurst exponent H	Roughness of a self-affine profile or memory of a time series; graph dimension $2 - H$	Topography, river flows, heartbeats, prices
Generalized dimensions D_q	A whole spectrum, weighting dense and sparse regions differently	Multifractals: turbulence, rainfall, galaxy clustering

For an exact self-similar fractal all the first six measures agree. For real data they can differ, which is itself informative: disagreement between measures often signals multifractality, self-affinity or a limited scaling range.

3.4 Recursion and iteration

Many fractals are generated by repeating an operation. The Koch curve repeats a geometric replacement. The Barnsley fern applies one of four affine maps at random, again and again, and the points converge onto the fern; Hutchinson's theorem guarantees that any set of contracting maps has exactly one such attractor. The Mandelbrot and Julia sets come from iterating the simple rule $z \rightarrow z^2 + c$ on complex numbers and asking which starting points stay bounded. Lindenmayer's L-systems rewrite strings of symbols in parallel, mimicking cell division, and can grow plant-like structures of striking realism.

Recursion explains why fractals can be complex yet informationally cheap. The entire Sierpiński triangle is specified by three points and one rule; the fern by 24 numbers. A small description that generates unbounded detail is precisely what a developing organism or an eroding landscape can 'implement'. But iteration does not always yield a fractal. Iterating a smooth contraction converges to a point; iterating many simple maps yields periodic orbits or chaos. Fractal structure appears when iteration combines contraction with branching or folding, so that structure is copied at ever smaller scales.

3.5 Multifractals

A single dimension describes where a structure is. It does not describe how *much* of something is there. Rainfall, turbulent energy dissipation, mineral concentrations and the distribution of galaxies are not just present or absent; they are concentrated very unevenly. A [multifractal](#) is a measure whose concentration varies so irregularly that different levels of intensity each occupy a fractal set with its own dimension. The description becomes a curve rather than a number: the *singularity spectrum* $f(\alpha)$, which gives the dimension of the set of points where the local intensity scales with exponent α .

Figure 8 shows the simplest example, a multiplicative cascade. Start with a unit of mass on an interval; split the interval in two and give 70 per cent of the mass to one half and 30 per cent to the other; repeat in each half, fourteen times. The result is a spiky, intermittent landscape in which the densest point holds about 20.7 times the average concentration. Its singularity strengths range from 0.523 to 1.729, and the set that carries most of the mass has dimension 0.881, although the support is the full interval. This model, with a split near 0.7, was proposed by [Meneveau and Sreenivasan](#) as a description of energy dissipation in turbulence and fits measurements well. The lesson is that many natural systems are not fractal in the simple sense; they are multifractal, and a single dimension misdescribes them.

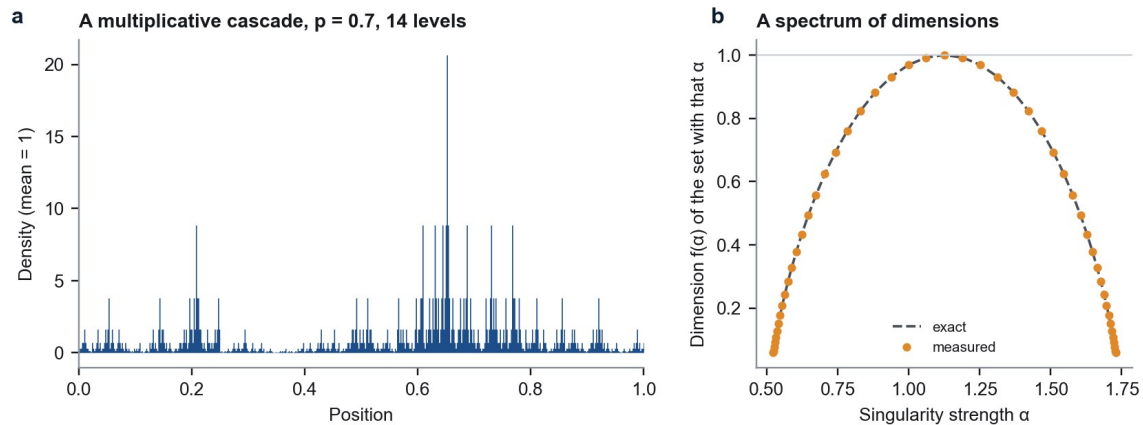


Figure 8. A multifractal cascade. (a) Mass split 70:30 at each of 14 levels, with the larger share assigned at random, produces an intermittent landscape of spikes. (b) Its singularity spectrum, measured from the model, matches the exact curve; a simple fractal would be a single point.

Source: H Heuristics calculation. Binomial p -model with $p = 0.7$, 16,384 cells (displayed at 512); $\tau(q)$ estimated by coarse-graining over 2–10 levels and Legendre-transformed.

3.6 Cutoffs, scaling ranges and what 'fractal' means in practice

Mathematical fractals have detail at every scale. Physical ones do not. A coastline is rough down to the grain of sand and up to the size of the continent, but not beyond. A bronchial tree branches from the trachea to the alveoli and then stops. Every physical fractal therefore has an *inner cutoff* and an *outer cutoff*, and the ratio between them, usually expressed in powers of ten or *decades*, is its scaling range. A claim that something is fractal is a claim about that range. One decade of scaling is a weak claim; it can be produced by structures that are not fractal at all (Section 11). Three or more decades is a strong one. The report therefore records scaling ranges wherever the literature allows, and treats 'fractal' in physical contexts as shorthand for 'statistically self-similar or self-affine over a stated range of scales'.

Key point. A fractal in mathematics is an object; a fractal in nature is a claim about a range of scales. The claim is only as strong as the range, the fit and the alternatives that were tested.

04

THE THEORETICAL FRAMEWORK



4. The Theoretical Framework

4.1 The question

Research question. How can fractal claims from very different domains be compared, graded and explained within one framework?

The Architecture of Creation framework describes form as the outcome of three ingredients: constraints that define what is possible, generative processes that explore possibilities, and filters that determine what persists. This section translates that framework into fractal terms. It identifies the generative processes that produce fractal structure, offers a ladder for grading claims, and sets out the tests that a claim of shared fractal architecture must pass.

4.2 Seven generative mechanisms

Surveying the domains covered in Sections 5 to 8, the report identifies seven processes that account for nearly all well-documented fractal structure (Figure 9, Table 4). They are not mutually exclusive; real systems often combine two or three.

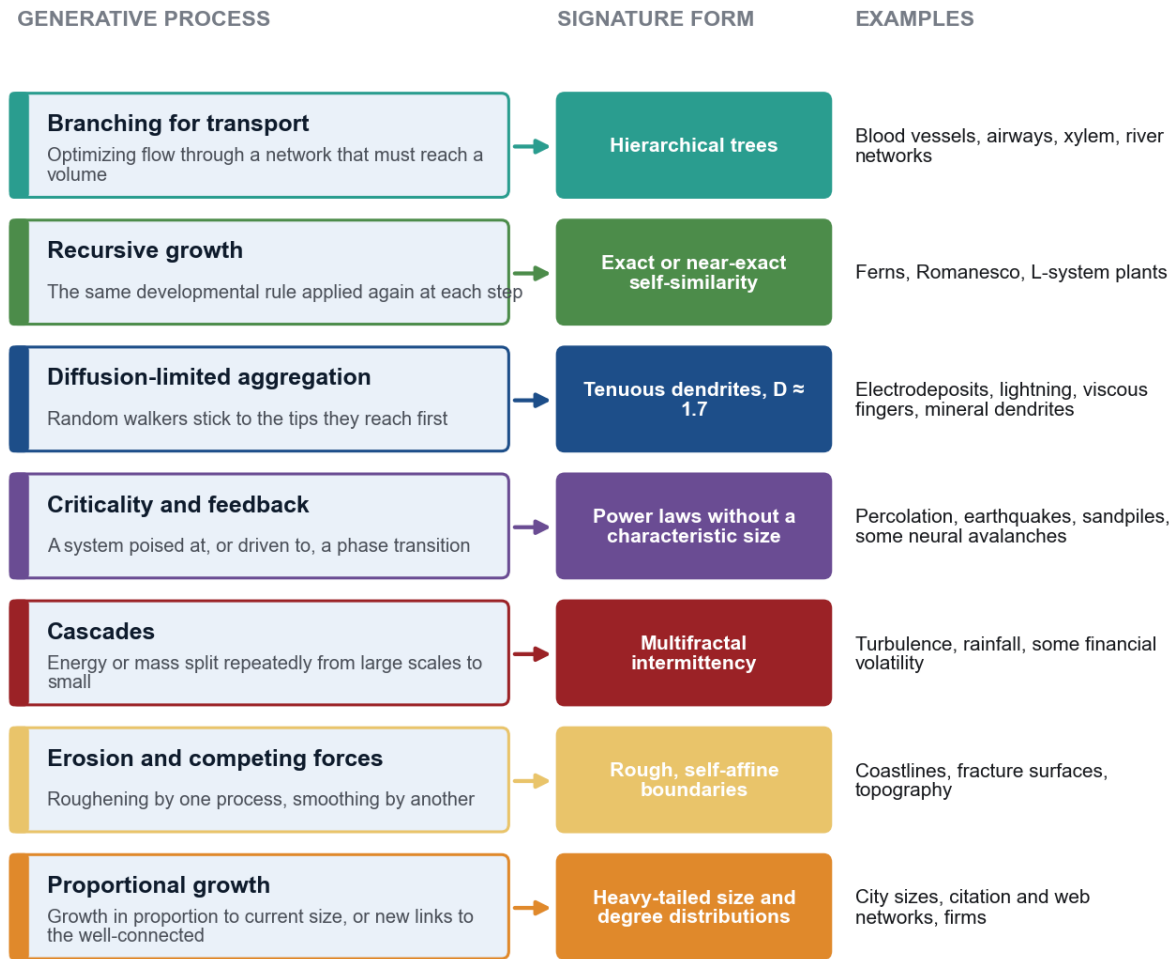


Figure 9. Seven generative mechanisms of fractal structure, the signature form each tends to produce, and examples. The mechanisms are distinct physical processes; the forms they produce can look alike.

Source: H Heuristics synthesis of the literature reviewed in Sections 2 and 5–8.

Table 4. Generative mechanisms and their quantitative signatures

Mechanism	Core rule	Quantitative signature that can be tested
Branching for transport	Minimize cost of moving a fluid through a network that must reach every part of a volume	Fixed radius and length ratios between generations; Murray's law $r_0^3 = \sum r_i^3$; Hack and Horton ratios
Recursive growth	Apply the same developmental rule at each step or scale	Exact or near-exact geometric ratios; discrete hierarchy of levels
Diffusion-limited aggregation	Growth fed by a diffusing field, fastest at protruding tips	Mass–radius dimension ≈ 1.71 (2D) or ≈ 2.5 (3D)
Criticality	System at or driven to a threshold between order and disorder	Power-law event sizes with exponents set by the universality class; finite-size cutoffs
Cascades	Energy or mass split repeatedly from large to small scales	Multifractal spectrum; Kolmogorov-type spectral slopes

Mechanism	Core rule	Quantitative signature that can be tested
Erosion and competing forces	One process roughens a surface; another smooths it	Self-affine roughness with Hurst exponent H ; scale-dependent dimension
Proportional growth	Growth or new links in proportion to existing size or connectivity	Heavy-tailed size or degree distributions; Zipf exponent near 1

A mechanism is supported for a given system when its signature, not merely its appearance, is found. Signatures are the bridge between geometry and cause.

4.3 A ladder of fractal claims

Not all fractal claims are equal. The report grades them on a five-rung ladder (Figure 10). At rung 0 is visual resemblance, which is not evidence at all. At rung 1 is a straight line on a log-log plot over about one decade. At rung 2 is robust scaling over two or more decades, with cutoffs stated and alternatives tested. At rung 3 a generative mechanism has been identified that reproduces the scaling and its cutoffs. At rung 4, different systems share exponents because they share a mechanism class: this is universality, and it is the only rung at which fractal similarity between different systems can be said to reveal a shared principle.

stronger claim, more evidence required

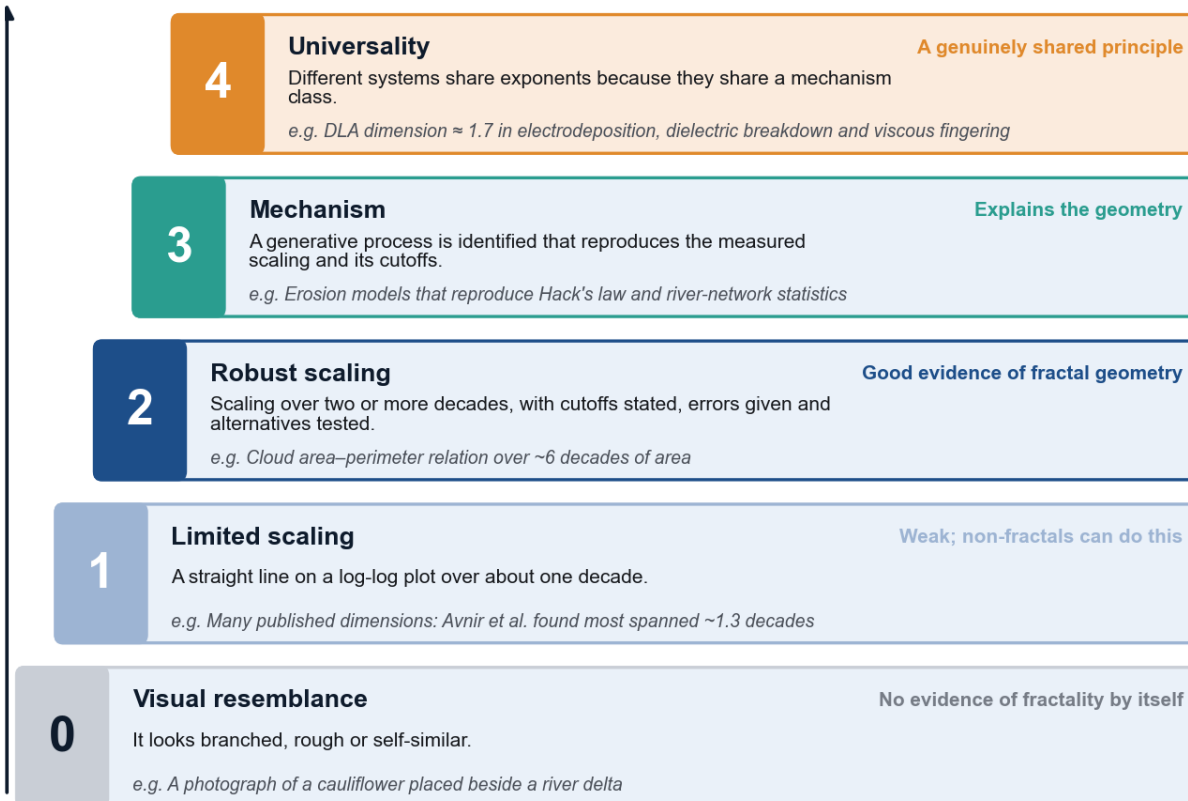


Figure 10. A ladder of fractal claims. Each rung requires more evidence than the one below and licenses a stronger conclusion. Only at rung 4 does shared geometry between different systems imply a shared principle.

Source: H Heuristics. Examples from Avnir et al. (1998), Lovejoy (1982), Rinaldo et al. (2014) and the DLA literature cited in Section 9.

4.4 Five tests for a claim of shared fractal architecture

The central question of the report, whether fractal similarity between domains reveals a shared principle, can be turned into tests. A claim that two systems share a fractal architecture should pass all five in Table 5.

Table 5. Five tests for a claim that two systems share a fractal architecture

Test	Question	Failure looks like
1. Range	Does each system scale over at least two decades, with cutoffs identified?	Scaling over one decade or less; cutoffs ignored
2. Statistics	Is the fractal or power-law model preferred over plausible alternatives (lognormal, exponential cutoff, crossover)?	A straight line on a log-log plot with no comparison of models
3. Numbers	Do the systems share exponents or spectra, not just the label 'fractal'?	Dimensions that differ, or that agree only within large error bars
4. Mechanism	Is there a generative process that, applied to each system, reproduces its measured scaling?	A mechanism invoked by analogy but not modelled
5. Texture	Do measures beyond dimension (lacunarity, multifractal spectrum, branching ratios) also agree?	Same dimension but different gappiness, anisotropy or topology

Systems that pass all five share a fractal architecture in the strong sense. Systems that pass tests 1–2 for each but fail 3–5 share a descriptive language but not, on present evidence, a cause.

4.5 Formal statement of the hypothesis

Combining the pieces, the report evaluates the Fractal Architecture Hypothesis in four parts:

- H1 (prevalence).** Fractal or multifractal structure over two or more decades of scale occurs in each of the four domains examined: life, Earth, the universe and civilization.
- H2 (mechanism).** In each domain, documented fractal structure can be traced to one or more of the seven generative mechanisms.
- H3 (tracking).** Fractal similarity between systems tracks similarity of mechanism: systems sharing a mechanism class share exponents and spectra, and systems with different mechanisms differ on at least one of tests 3–5.
- H4 (boundedness).** Every physical fractal has finite cutoffs set by identifiable physics, and no domain, including the universe as a whole, is fractal at all scales.

H1 and H2 are the physical and biological level of the hypothesis. H3 is the bridge to the foundational level: if it holds, fractal geometry is not merely a common description but a guide to common causes. H4 is a constraint that any foundational claim must respect. Sections 5 to 11 gather the evidence; Section 12 tests H3 and H4 directly.

05

DOMAIN I · LIFE

5. Life and Biological Organization

5.1 The question

Research question. Where is fractal structure genuine in living systems, what generates it, and does it explain biological scaling laws?

Living systems are the domain in which fractal language is most often used and most often misused. Organisms must move fluids, gases, nutrients and signals through volumes, and they build the necessary networks by repeating developmental rules. Both facts favour branching hierarchies. The questions are how close those hierarchies come to true self-similarity, over how many levels, and whether their geometry explains the famous scaling laws of biology.

5.2 Vascular networks and Murray's law

The circulatory system is the archetypal biological fractal. From the aorta, about 1.25 centimetres in radius in an adult human, to capillaries a few micrometres across, arteries divide through roughly 20 to 30 generations, spanning more than three decades of radius. At each junction the radii of parent and daughter vessels are related in a regular way. Murray showed in 1926 that if a vessel must carry flow Q , the total cost of pumping blood through it (which falls steeply as radius grows, as $1/r^4$) and of maintaining the blood and vessel wall (which rises as r^2) is minimized when r^3 is proportional to Q (Figure 11a). Because flow is conserved at a junction, the optimum requires the cube of the parent radius to equal the sum of the cubes of the daughter radii. This is [Murray's law](#), and it holds well in small arteries and arterioles. In the largest arteries, where flow is pulsatile, the ratio is closer to *area-preserving*, $r_0^2 = \sum r_i^2$, which minimizes the reflection of pressure waves.

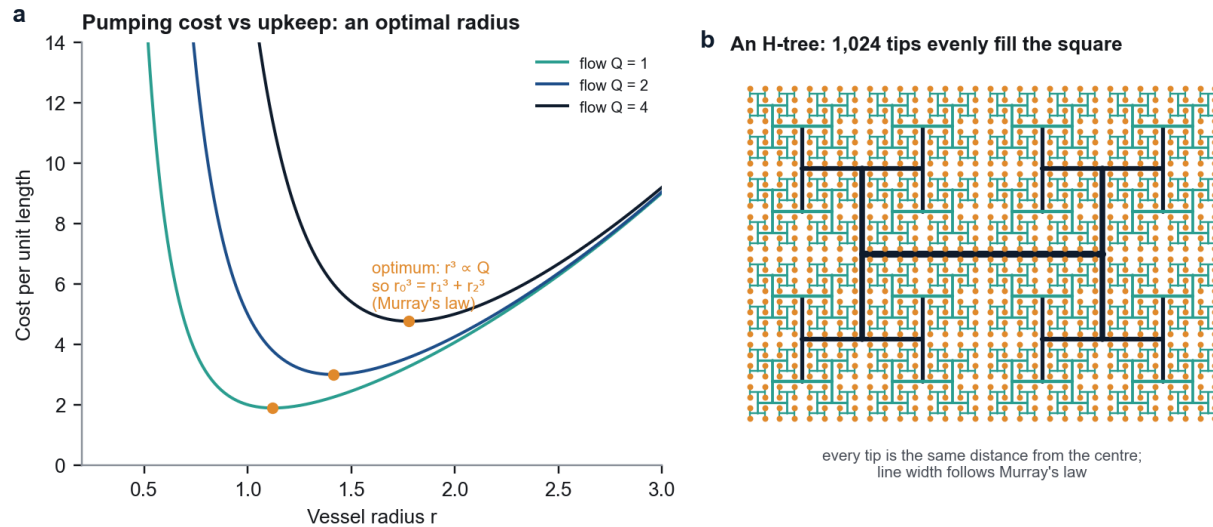


Figure 11. Why vascular trees branch as they do. (a) The cost of a vessel per unit length, pumping power plus upkeep, has a minimum at a radius whose cube is proportional to flow: Murray's law. (b) An H-tree, a hierarchy that delivers to 1,024 evenly spaced tips with every tip the same distance from the centre, drawn with line widths following Murray's law.

Source: H Heuristics calculation. $\text{Cost} = Q^2/r^4 + r^2$ in dimensionless units, so the optimum is $r = (2Q^2)^{1/6}$, giving $r^3 \propto Q$. H-tree of 10 levels with length ratio $1/\sqrt{2}$.

Vascular trees are therefore fractal-like in a precise, mechanistic sense: their branching ratios follow from an optimization principle, and measured trees in many organs and species show consistent scaling between vessel diameter, length and the volume they supply ([Huo and Kassab 2012](#)). The retinal vasculature, visible directly through the pupil, has a box-counting dimension near 1.7 in healthy eyes, and departures from it have been studied as markers of disease ([Masters 2004](#)). The fractal description, however, holds only over the range in which the branching rule is constant. The aorta and the capillary bed are each governed by different physics, and the transition between pulsatile and smooth flow changes the scaling ratio partway down the tree.

Assessment. Vascular trees are fractal-like over about three decades of radius, with branching ratios explained by a cost-minimization principle (Grade B). They are hierarchies built by an optimizing rule, not ideal self-similar objects.

5.3 The lung: optimal, and deliberately not quite

The human bronchial tree divides about 23 times between the trachea and the alveolar sacs, almost always into two. Weibel's morphometry showed that airway diameters shrink by a nearly constant factor from one generation to the next over the first dozen or so generations ([Weibel and Gomez 1962](#)), and that the result packs a gas-exchange surface on the order of a hundred square metres into a chest of a few litres. West, Bhargava and Goldberger showed that the diameter data are better described by a fractal scaling with a superimposed modulation than by a single exponential ([West et al. 1986](#)).

The ratio of daughter to parent diameter in the conducting airways is close to $2^{-1/3} \approx 0.79$, which is what Murray's law predicts for symmetric branching. But Mauroy and colleagues found that it is slightly larger than the value that would minimize the work of breathing, and showed why: in an exactly optimal tree, a small constriction of the airways, as in asthma, would cause a disproportionate rise in resistance. The real lung trades a little efficiency for a safety margin ([Mauroy et al. 2004](#)). This is an important result for the

report's argument. The lung's near-fractal geometry is the outcome of optimization *under* constraints, including robustness, not the expression of an abstract preference for self-similarity.

5.4 Metabolic scaling and the 3/4 law

Kleiber's observation that metabolic rate B scales with body mass M as roughly $M^{3/4}$ has been called the most general scaling law in biology. Geometric similarity predicts $M^{2/3}$, because surface area scales as mass to the $2/3$ power. In 1997 West, Brown and Enquist proposed that the extra $1/12$ comes from the fractal geometry of distribution networks. If a network is space-filling, has terminal units (capillaries) of fixed size, and minimizes the energy needed to pump fluid through it, then blood volume scales as the number of capillaries to the $4/3$ power, and metabolic rate, set by the number of capillaries, scales as mass to the $3/4$ power ([West et al. 1997](#)).

The theory is elegant and influential. It is also contested on three fronts. Empirically, the best-fitting exponent for mammalian basal metabolism has been estimated nearer $2/3$ than $3/4$ in some analyses ([White and Seymour 2003](#); [Dodds et al. 2001](#)), and the log-log relation shows systematic curvature, with a steeper slope in large mammals ([Kolokotronis et al. 2010](#)). Theoretically, the derivation depends on assumptions about how vessel radius changes down the tree that hold only in parts of real networks. And mathematically, the $3/4$ exponent emerges only in the limit of infinitely many branching levels ([Savage et al. 2008](#)).

Figure 12 makes the last point concrete. The report built model networks with between 4 and 40 levels of symmetric binary branching, space-filling length ratios and two rules for radius: area-preserving throughout, or area-preserving in large vessels with Murray's law in the eight smallest levels, as in real circulatory systems. For each network it computed blood volume and number of capillaries, and from neighbouring network sizes the local scaling exponent. With area-preserving branching the exponent was 0.739 at 12 levels and 0.748 at 20, approaching $3/4$ from below. With Murray's law in the smaller vessels it was 0.806 at 12 levels and 0.761 at 20, approaching from above. Both reach 0.75 only at around 30 levels or more. Mammals span roughly 15 to 30 levels between the smallest shrews and the largest whales, so the model itself predicts that the observed exponent should drift with body size rather than equal $3/4$ everywhere.

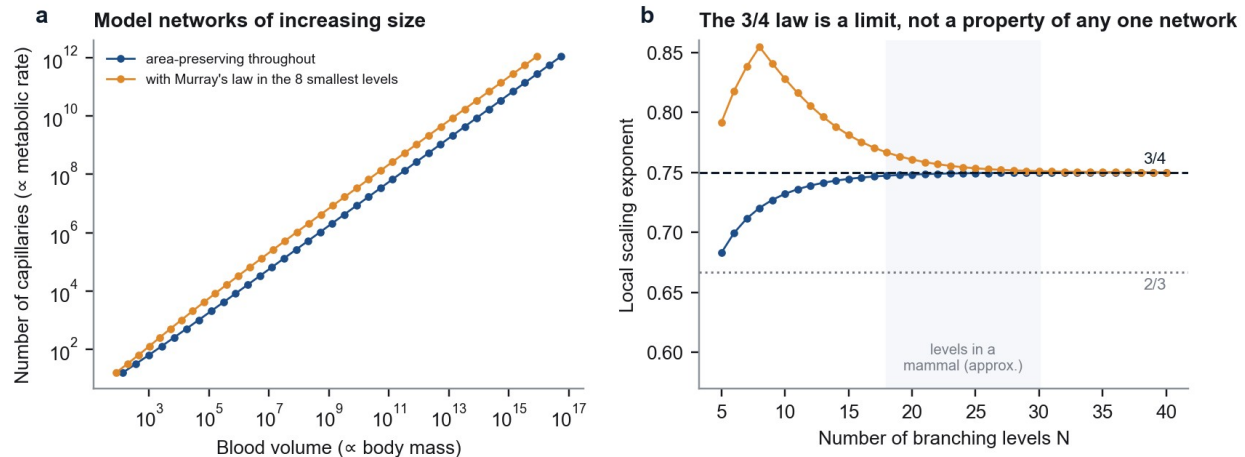


Figure 12. The 3/4 law is a limit, not a property of any one network. (a) Number of capillaries (a proxy for metabolic rate) against blood volume (a proxy for body mass) for model networks of 4 to 40 levels. (b) The local scaling exponent between neighbouring network sizes. The exponent approaches 3/4 from below with area-preserving branching and from above when Murray's law governs the smallest vessels.

Source: H Heuristics calculation. Symmetric binary networks; length ratio $2^{-1/3}$; radius ratio $2^{-1/2}$ (area-preserving) or $2^{-1/3}$ (Murray) in the eight levels nearest the capillaries; capillary radius and length fixed. Shaded band: approximate range of levels in mammals.

Assessment. That distribution networks shape metabolic scaling is plausible and widely accepted. That an ideal fractal network explains a universal 3/4 exponent is contested (Grade C): data and finite-size models both point to a size-dependent exponent.

5.5 Plants: Leonardo's rule and recursive growth

Leonardo da Vinci noted that when a tree branches, the combined thickness of the branches equals the thickness of the trunk below them. This [Leonardo rule](#) of area preservation holds remarkably well in many species. Christophe Eloy showed that it follows if a tree's branches are built to resist wind-induced breakage equally at every level, and that it implies a self-similar branching geometry. West, Brown and Enquist extended their network model to the vascular systems of plants ([West et al. 1999](#)). Plant architecture is also the clearest case of *recursive growth*: a shoot meristem produces leaves and buds by a repeated rule, and the buds become new shoots that repeat it. L-systems capture this, and with a few lines of rewriting rules can generate convincing trees, ferns and inflorescences.

The Romanesco cauliflower is often presented as nature's most perfect fractal: a cone of spirals, each composed of smaller spiral cones, repeated over several levels. In 2021 Azpeitia and colleagues identified how it arises ([Azpeitia et al. 2021](#)). Mutations in floral-identity genes leave each would-be flower stuck in an inflorescence state, so that instead of becoming a flower it produces more inflorescences, which in turn fail to become flowers. Phyllotaxis arranges them in golden-angle spirals; a gradual change in meristem size during growth turns the spirals into cones. The Romanesco is fractal because a developmental program is caught in a loop, a striking example of recursion made literal. It is also limited: the self-similarity extends over a handful of levels before the structures become too small to subdivide.

5.6 Neurons, brains and genomes

Neurons are among the most elaborately branched cells. Their dendritic arbors have measurable fractal dimensions, typically between about 1.2 and 1.8 in two-dimensional projections, which vary by cell type, species and developmental stage ([Smith et al. 1996](#)). Cuntz and colleagues showed that a single rule,

connecting synaptic targets with a tree that balances total wiring length against conduction path length, reproduces the shapes of many neuron types (Cuntz et al. 2010). That is a transport-optimization mechanism of the same family as Murray's law. The folded surface of the cerebral cortex has also been described as fractal, though whether it is truly self-similar or simply folded over a narrow range of scales remains debated (Kiselev et al. 2003). Across mammals, cortical folding follows a single scaling law linking exposed surface, total surface and thickness, consistent with a physical folding mechanism (Mota and Herculano-Houzel 2015).

Inside the nucleus, the genome is packed into a space a million times shorter than its length. In 2009 the first genome-wide maps of chromosomal contacts found that the probability of contact between two loci falls with their separation along the chromosome roughly as one over the separation, over a range of about half a megabase to several megabases. This matched a *fractal globule*, a knot-free, hierarchically crumpled polymer (Lieberman-Aiden et al. 2009; Mirny 2011). Later work has refined the picture with active loop extrusion and compartments, so the fractal globule is best regarded as an approximate description of one regime of chromatin organization (Grade C).

5.7 Fractal dynamics: heartbeats and long memory

Fractal structure can exist in time as well as space. The intervals between healthy heartbeats are not regular: they fluctuate in a way that is statistically self-similar over scales from seconds to hours. Goldberger, Peng and colleagues developed [detrended fluctuation analysis](#) (DFA) to measure such correlations, and found that healthy heart rhythms have a DFA exponent near 1, characteristic of $1/f$ noise, whereas heart failure and ageing push the exponent away from 1 (Goldberger et al. 2002). Healthy heartbeat dynamics are also multifractal, and lose multifractality in disease (Ivanov et al. 1999).

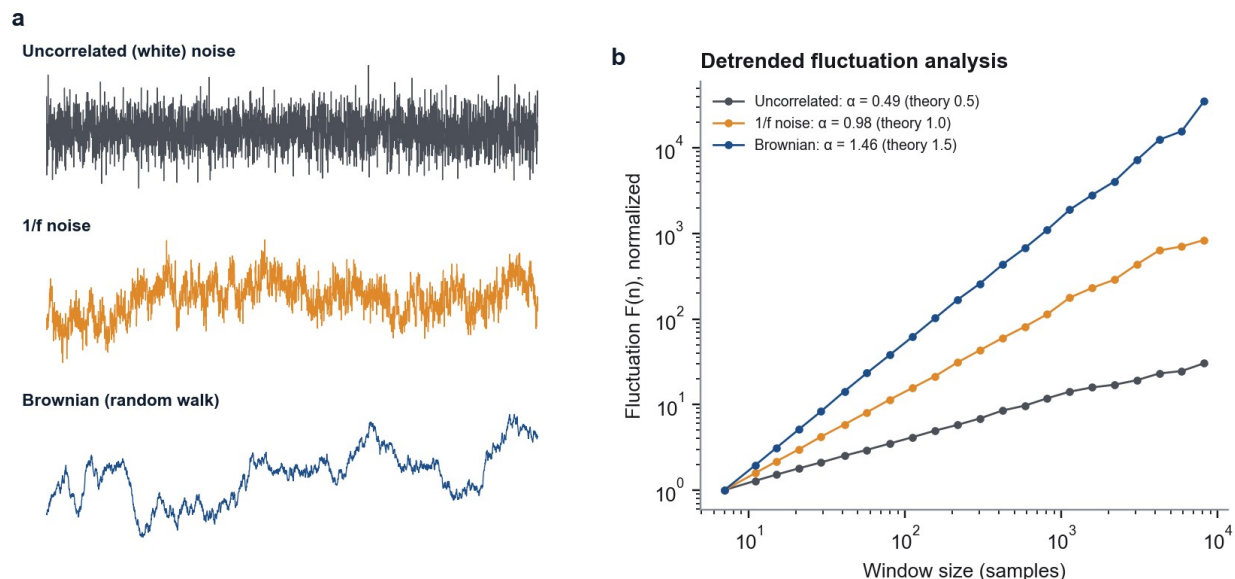


Figure 13. Measuring fractal dynamics. (a) Segments of three synthetic signals: uncorrelated noise, $1/f$ noise and a random walk. (b) Detrended fluctuation analysis recovers their scaling exponents, 0.5, 1.0 and 1.5. Healthy heartbeat intervals typically show exponents near 1; departures toward 0.5 or 1.5 are associated with disease.

Source: H Heuristics calculation. 65,536-sample signals generated by spectral synthesis; DFA with linear detrending over 22 window sizes from 8 to 8,192 samples. The signals are synthetic illustrations, not physiological data.

Figure 13 shows the method on synthetic signals of known character. DFA returned 0.485 for white noise, 0.983 for $1/f$ noise and 1.458 for a random walk, against theoretical values of 0.5, 1.0 and 1.5. The

interpretation of fractal physiology is that healthy systems are regulated by many interacting feedback loops operating at different time scales, producing correlations without a dominant period. That is plausible and clinically useful, but the precise mechanism is not established, and the same exponent can arise from different underlying dynamics.

5.8 A cautionary case: Lévy flights

Not every biological fractal claim has survived. In the 1990s it was reported that wandering albatrosses, and later deer and bumblebees, search for food using [Lévy flights](#), random walks with power-law step lengths that generate fractal search patterns and are theoretically efficient for sparse targets. A re-analysis found that the albatross result arose from misinterpreting the time birds spent sitting on the water, and that with correct data and proper statistical comparison the evidence for Lévy flights in these species disappeared ([Edwards et al. 2007](#)). The idea is not dead, but the episode illustrates the pattern of the critical turn: a striking fractal claim based on a log-log plot weakened under rigorous testing.

5.9 Assessment

Table 6. Fractal claims in living systems, graded

Claim	Evidence	Mechanism	Grade
Arterial and venous trees are fractal-like	Consistent branching ratios over ~3 decades of radius in many organs and species	Transport optimization (Murray's law)	B
The bronchial tree is fractal-like	~23 generations with near-constant diameter ratio in conducting airways	Optimization with a safety margin	B
Tree branching follows Leonardo's rule	Area preservation measured in many species	Mechanical design against wind	B
Romanesco is a recursive fractal	Several levels of spiral cones; genetic mechanism identified	Recursive growth from a stalled program	B
Metabolic rate $\propto M^{3/4}$ because networks are fractal	Exponent and curvature disputed; finite-size effects	Network model contested	C
Neuronal arbors are fractal	Measurable but cell-type-specific dimensions over limited ranges	Wiring optimization	C
Chromatin is a fractal globule	Contact scaling $\sim 1/s$ over about one decade; models refined since	Polymer crumpling, loop extrusion	C
Healthy heartbeats are fractal and multifractal	Replicated DFA and multifractal results; clinical associations	Multiscale feedback (not established)	B
Animals search by Lévy flights	Key results overturned or weakened by re-analysis	Disputed	C

Grades refer to the claim as stated in the first column over the range of scales indicated, not to whether the system is 'fractal' in an absolute sense.

Life provides strong support for H1 and H2: fractal-like structure is common, and the mechanisms, transport optimization and recursive growth, are identified. It also adds an essential nuance. Biological fractals are *engineered by selection*: their ratios are tuned to costs and risks, they change rule partway down the hierarchy, and they stop where function stops. The $3/4$ law shows the danger of moving from 'fractal-like over some levels' to 'ideally fractal, therefore a universal exponent'.

06

DOMAIN II · EARTH



6. Earth and Natural Systems

6.1 The question

Research question. Which features of Earth's surface, atmosphere and crust are fractal, over what ranges, and what physical processes make them so?

Earth is where fractal geometry began as an empirical science, and where it has been tested most thoroughly. Coastlines, river networks, clouds, turbulent flows, mountain ranges, fault systems and earthquake catalogues all show scaling over wide ranges. Unlike living systems, these structures are not shaped by natural selection. Their form comes from the competition of physical processes: flow and erosion, uplift and weathering, stress and fracture. That makes Earth an ideal test of the claim that fractal structure arises from a few generic mechanisms.

Each subsection below pairs a natural system with a calculation made for this report, in which a simple process is run and its fractal statistics measured. The calculations are not simulations of Earth. They test whether the proposed mechanism can, in principle, produce the observed scaling.

6.2 Coastlines

Richardson's measurements gave dimensions of about 1.02 for the smooth coast of South Africa, 1.13 for Australia, 1.15 for the land frontier of Germany and 1.25 for the deeply indented west coast of Britain, as Mandelbrot reported in 1967. The value expresses how rapidly measured length grows as the ruler shrinks: with dimension D , length scales as ϵ^{1-D} . Figure 14 repeats Richardson's experiment on four random Koch curves built to have dimensions of 1.02, 1.13, 1.25 and 1.45. Walking each with dividers of 12 openings between 0.001 and 0.1 of the coast's span recovered dimensions of 1.022, 1.136, 1.237 and 1.441. For the roughest curve, shrinking the ruler a hundredfold increased the measured length about eightfold.

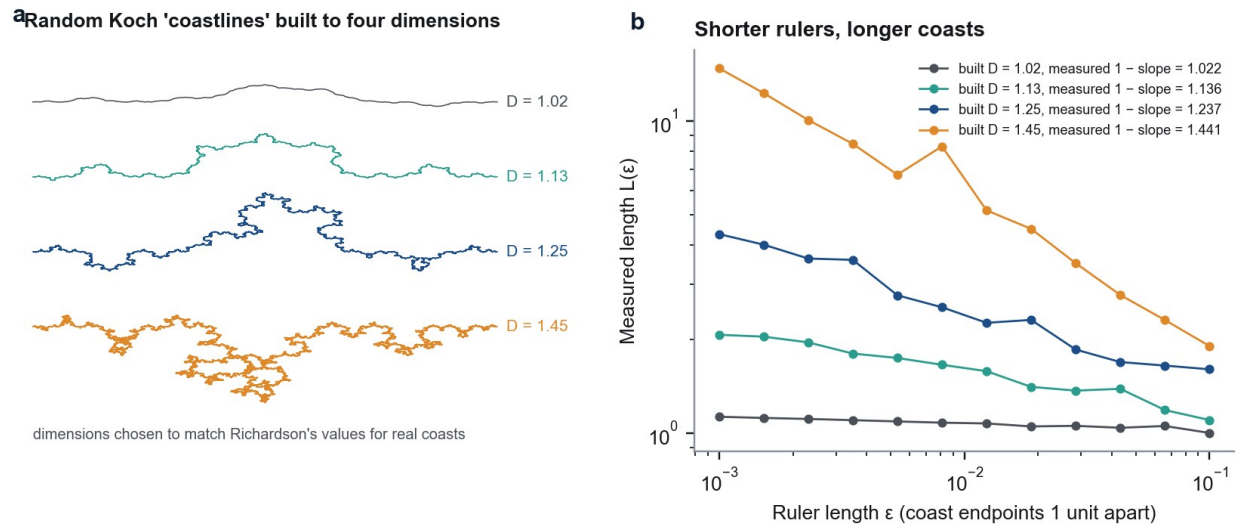


Figure 14. Richardson's coastline effect. (a) Random Koch curves built to four dimensions chosen to match Richardson's values for real coasts, plus one rougher curve. (b) Measured length against ruler length; the slope gives $1 - D$. The divider method recovers each built-in dimension to within about 0.02.

Source: H Heuristics calculation. Koch curves of 8 levels with random bump orientation and bump angle set so that $D = \log 4 / \log(2 + 2 \cos \theta)$; divider walks at 12 ruler lengths from 0.001 to 0.1.

Why should coasts be fractal? Bernard Sapoval and colleagues offered a mechanism: waves erode a rocky coast fastest where it is weakest and most exposed. As the coast becomes more irregular, its bays and headlands dissipate wave energy, slowing erosion until the coastline stabilizes in a fractal shape (Sapoval et al. 2004). In their model the stabilized coast has a dimension close to $4/3$, the value for the accessible outer perimeter of a percolation cluster. That is an erosion-and-competing-forces mechanism, and it links coastlines to the universality class of percolation. Real coastlines also reflect geology, sea-level history and sediment supply, so their dimensions vary from place to place and can change with the range of scales examined.

6.3 River networks

River networks are the most thoroughly quantified natural fractals. Horton's laws state that the number of streams of each order, their mean lengths and their drainage areas fall in constant ratios up the hierarchy, typically with a bifurcation ratio between 3 and 5. Hack's law states that main-stream length L grows with drainage area A as $L \propto A^h$, with h usually between about 0.5 and 0.6. Because h exceeds 0.5, larger basins are more elongated, and rivers meander, both signs of fractal geometry (Dodds and Rothman 2000). These laws hold from basins of a few square kilometres to continental drainage systems.

Figure 15 tests two candidate mechanisms. The first is the [Scheidegger model](#), in which water flows downhill along a random path on a tilted lattice: there is no landscape and no optimization, only random choices. It gave a Hack exponent of 0.645, close to its exact theoretical value of $2/3$. The second carved drainage into a fractal landscape: a tilted random surface with Hurst exponent 0.7, from which water was routed by flooding upward from the outlet so that depressions fill and spill as real lakes do. The result is the dendritic network in Figure 15a. Its Hack exponent was 0.558, inside the range of real basins.

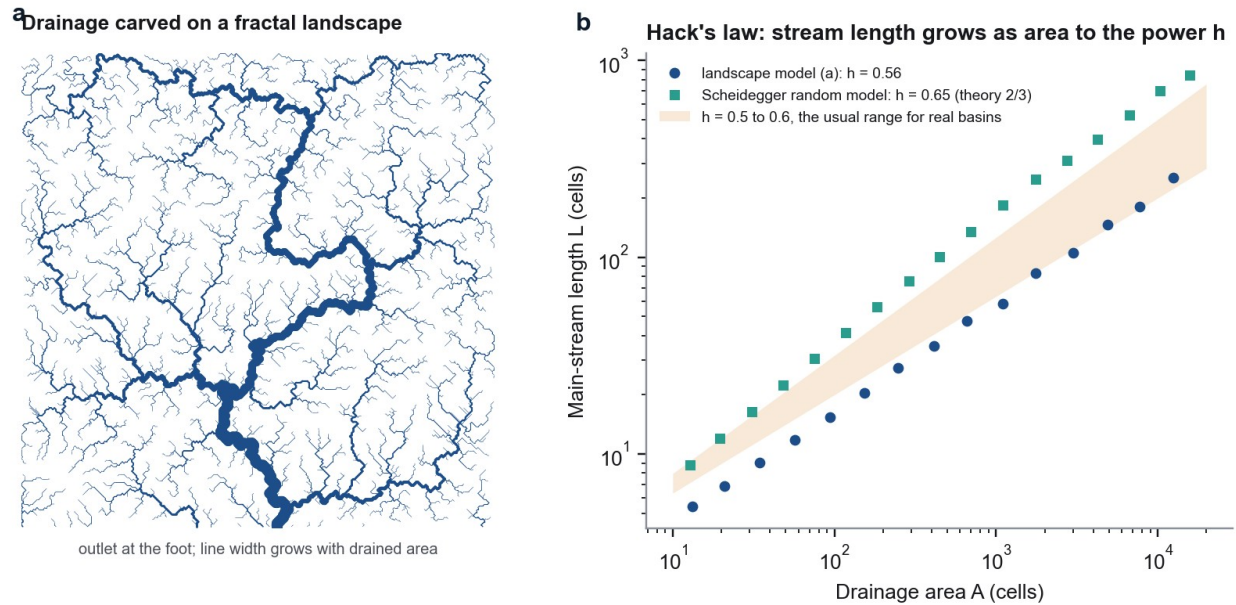


Figure 15. River networks and Hack's law. (a) Drainage carved into a tilted fractal landscape of 90,000 cells; line width grows with drained area. (b) Main-stream length against drainage area. The landscape model gives an exponent within the range of real basins; a purely random network gives a steeper exponent close to its theoretical $2/3$.

Source: H Heuristics calculation. Landscape: fractional Brownian surface ($H = 0.7$) plus a linear tilt, 300×300 cells, drainage by priority flooding with eight-neighbour routing. Scheidegger: $512 \times 1,024$ lattice with periodic sides. Exponents fitted on binned means.

The comparison carries a lesson that recurs throughout the report. Horton's ratios and Hack-like scaling are surprisingly easy to produce. James Kirchner showed that Horton's laws are a statistical near-inevitability of almost any branching network ([Kirchner 1993](#)), and Shreve's random topology model reproduces them ([Shreve 1966](#)). The exponents therefore cannot by themselves identify a mechanism. What distinguishes real rivers is that they also minimize energy dissipation. Rinaldo, Rodríguez-Iturbe and colleagues showed that networks that minimize total energy expenditure, *optimal channel networks*, reproduce the full set of observed statistics, including the distribution of drainage areas and the three-dimensional form of basins ([Rinaldo et al. 1992](#); [Rodríguez-Iturbe et al. 1992](#); [Rinaldo et al. 2014](#)). River networks thus combine two mechanisms: erosion and competing forces build them, and the result approximates a transport optimum. That puts them in the same family as vascular trees, a point Section 9 examines.

6.4 Clouds and rain

In 1982 Shaun Lovejoy measured the areas and perimeters of cloud and rain regions in satellite and radar images spanning areas from 1 to 1.2 million square kilometres. If the regions were smooth, perimeter would grow as the square root of area. Instead it grew as area to the power $D/2$ with $D \approx 1.35$, across the whole range ([Lovejoy 1982](#)). Cloud boundaries are fractal over three decades of length, one of the widest scaling ranges known for any natural object.

The atmosphere's turbulent mixing generates scalar fields, such as humidity, that are rough at all scales in the inertial range. A cloud is roughly the region where such a field exceeds a threshold. Figure 16 tests whether that alone can produce the observed geometry. The report generated a fractional Gaussian surface with Hurst exponent 0.3, cut it at a threshold, and measured the area and perimeter of each resulting 'cloud', with holes filled so that area is the area enclosed by each perimeter. Across 628 clouds

the relation gave $D = 1.34$, matching the theoretical prediction for contour loops of such surfaces, $(3 - H)/2 = 1.35$ ([Kondev and Henley 1995](#)), and Lovejoy's observation.

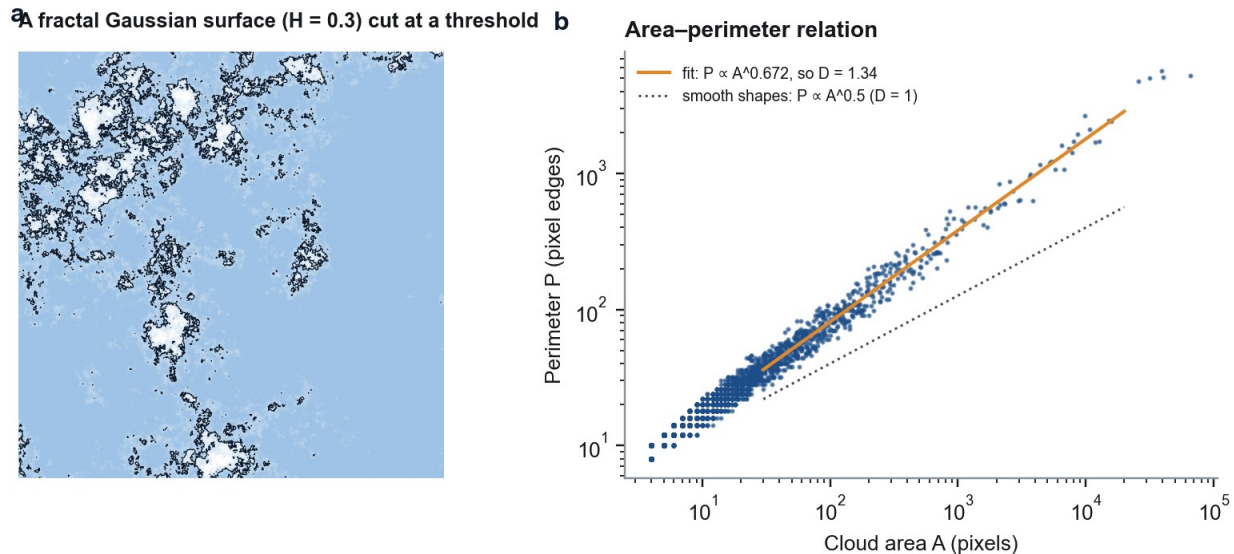


Figure 16. Clouds from a fractal field. (a) A fractional Gaussian surface with Hurst exponent 0.3, thresholded to produce 'clouds' (white) against sky. (b) Perimeter against area for the resulting clouds. The fitted dimension matches both the theoretical value for contour loops and Lovejoy's measurement for real cloud and rain areas.

Source: H Heuristics calculation. $2,048 \times 2,048$ spectral-synthesis surface; threshold 1.2 standard deviations; holes filled; perimeter counted as pixel edges; fit over areas of 30 to 20,000 pixels.

The agreement is striking but must be read carefully. It shows that the observed dimension is what a thresholded field with $H \approx 0.3$ produces. It does not show that clouds *are* thresholded Gaussian fields; real atmospheric fields are multifractal and anisotropic, being stratified by gravity. Lovejoy and Schertzer's later work develops multifractal cascade models of the atmosphere that are more realistic and more complex.

6.5 Turbulence

Turbulence is the oldest physical cascade. Energy fed into a fluid at large scales passes to smaller and smaller eddies until viscosity dissipates it as heat. Kolmogorov's 1941 theory predicted that in the intermediate *inertial range*, the energy spectrum falls as wavenumber to the $-5/3$ power, one of the best-confirmed predictions in fluid dynamics. Measurements soon showed, however, that dissipation is not spread evenly: it is concentrated in intense, intermittent bursts. Kolmogorov and Obukhov revised the theory in 1962 to allow for this ([Obukhov 1962](#)), and later work described the dissipation field as a multifractal ([Meneveau and Sreenivasan 1987](#)), exactly the kind of measure shown in Figure 8. The scaling range in the atmosphere spans from millimetres to hundreds of kilometres. Turbulence is therefore a well-established multifractal (Grade A for the existence of multiscaling; the precise spectrum remains under study).

6.6 Topography, fractures and earthquakes

Landscapes are self-affine over wide ranges: elevation profiles look statistically similar when horizontal distances are stretched by one factor and heights by another. Fractal terrain generated by fractional Brownian surfaces is the basis of computer-graphics landscapes. Real topography, however, has a

dimension that changes with scale, reflecting different dominant processes at different scales, such as tectonics, fluvial erosion and hillslope creep ([Mark and Aronson 1984](#)). Rock fragments produced by crushing and blasting follow power-law size distributions with fractal dimensions typically between 2 and 3 ([Turcotte 1986](#)), and fault and fracture systems show scaling of length and spacing over several decades, though with important exceptions and characteristic scales set by layering ([Bonnet et al. 2001](#)).

The [Gutenberg–Richter law](#) states that the number of earthquakes larger than magnitude m falls by a factor of about ten for each unit increase in magnitude. Because magnitude is logarithmic in energy, this is a power law in released energy, with no characteristic earthquake size between the smallest detectable tremors and the great ruptures. Self-organized criticality offers an explanation: the crust is slowly loaded by plate motion and releases stress in avalanches on a network of faults poised near failure ([Bak et al. 1987](#)).

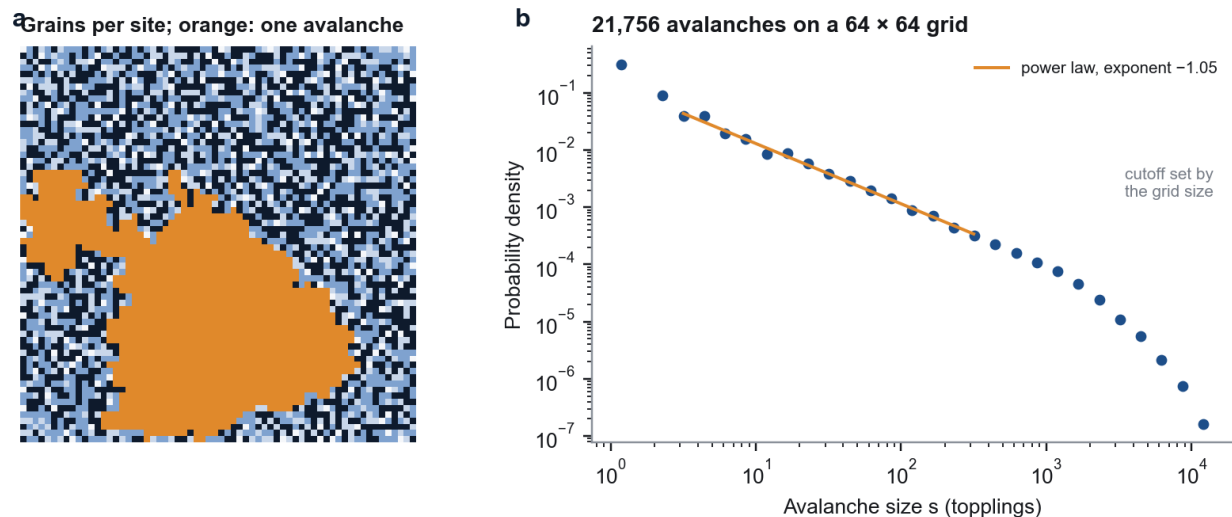


Figure 17. Self-organized criticality. (a) A Bak–Tang–Wiesenfeld sandpile after many grains have been added: shades show grains per site; orange marks the sites toppled in one large avalanche. (b) The distribution of avalanche sizes is a power law over about two decades before a cutoff set by the grid size; no parameter was tuned to produce it.

Source: H Heuristics calculation. 64×64 grid with open boundaries; grains added at random sites; sites with four or more grains topple to their neighbours. 60,000 grains, the first 10,000 discarded; power law fitted for sizes 3–300 topplings.

Figure 17 runs the original sandpile model. Grains are dropped at random on a grid; any site holding four grains topples, passing one to each neighbour, which may topple in turn. No parameter is adjusted. After a transient the pile settles into a state in which avalanches of every size occur: across 21,756 avalanches, sizes ranged from a single toppling to 14,217, and their distribution followed a power law with exponent about 1.05 over two decades before a cutoff set by the size of the grid. This is the essence of the self-organized criticality hypothesis: fractal statistics as the natural end state of slowly driven systems with thresholds. Whether real earthquakes, forest fires or extinctions are self-organized critical, rather than merely heavy-tailed for other reasons, is debated case by case (Grade C as a general explanation; Grade B for the Gutenberg–Richter law itself).

6.7 Lightning, minerals and porous rocks

Several Earth phenomena are produced by diffusion-limited growth. Lightning and other electrical discharges propagate fastest where the electric field is strongest, at the tips of existing channels, producing branched patterns whose dimension matches the dielectric-breakdown model, a cousin of DLA

([Niemeyer et al. 1984](#)). Manganese oxide dendrites on rock surfaces, often mistaken for fossil plants, grow by the same process. When one fluid displaces another in porous rock, as when water pushes out oil, the invading fluid forms fractal fingers whose dimension is close to that of DLA ([Måløy et al. 1985](#)), and the pore spaces of sandstones are themselves fractal over a range of scales ([Katz and Thompson 1985](#)). These cases belong to a universality class discussed in Section 9.

6.8 Assessment

Table 7. Fractal claims in Earth systems, graded

System	Measured scaling	Approximate range	Mechanism	Grade
Coastlines	$D \approx 1.0\text{--}1.3$ (Richardson); varies by coast	~2 decades of ruler length	Erosion stabilized by dissipation	B
River networks	Hack $h \approx 0.5\text{--}0.6$; Horton ratios	Several decades of area	Erosion; energy minimization	B
Cloud and rain areas	$D \approx 1.35$ (area–perimeter)	~6 decades of area	Turbulent mixing, thresholding	B
Turbulence	$-5/3$ spectrum; multifractal dissipation	Millimetres to hundreds of km	Energy cascade	A
Topography	Self-affine; scale-dependent D	Varies by process regime	Uplift versus erosion	B
Fragments and fractures	Power-law sizes; $D \approx 2\text{--}3$	Several decades, with exceptions	Fragmentation; fracture growth	B
Earthquakes	Gutenberg–Richter power law	Many decades of energy	Possibly self-organized criticality	B (law); C (SOC)
Discharges, dendrites, fingering	$D \approx 1.7$ in 2D	2–3 decades in experiments	Diffusion-limited growth	A

Ranges are indicative and depend on the study. Grades refer to the existence of the scaling over the stated range; claims about mechanism are graded in the text.

Earth supports H1 and H2 strongly. Its fractals are among the best measured in science, and for each there is at least one mechanism that reproduces the observed numbers. The domain also illustrates H4: every Earth fractal has cutoffs, from the grain size of sediment to the size of the planet, and many have dimensions that change from one range of scale to another as the dominant process changes.

07

DOMAIN III · THE UNIVERSE



7. The Universe

7.1 The question

Research question. Is the large-scale distribution of matter fractal, and if so, over what range of scales?

The distribution of matter in the universe forms a [cosmic web](#) of filaments, sheets, dense clusters at their intersections and vast voids between them. Images of the web, from surveys and simulations, are among the most fractal-looking pictures in science. The question of whether the universe is fractal is also of unusual importance, because modern cosmology rests on the [cosmological principle](#): that on large enough scales the universe is homogeneous and isotropic. A universe fractal at all scales would violate it.

7.2 A web built by gravity

The cosmic web grew from tiny density fluctuations in the early universe, measured today in the cosmic microwave background. Those fluctuations were nearly scale-invariant: their amplitude was almost the same on every scale, a property predicted by cosmic inflation and confirmed to high precision. Gravity amplified them. Slightly denser regions attracted matter from their surroundings and collapsed, first along one axis into sheets, then into filaments, then into compact haloes where galaxies formed. Yakov Zel'dovich described the first stage with a simple approximation in 1970, and Bond, Kofman and Pogosyan showed how filaments are 'woven' between the peaks of the initial density field ([Bond et al. 1996](#)). Large simulations such as the Millennium Run reproduce the observed web in detail ([Springel et al. 2005](#)), and many algorithms now classify the web into its components ([Libeskind et al. 2018](#)).

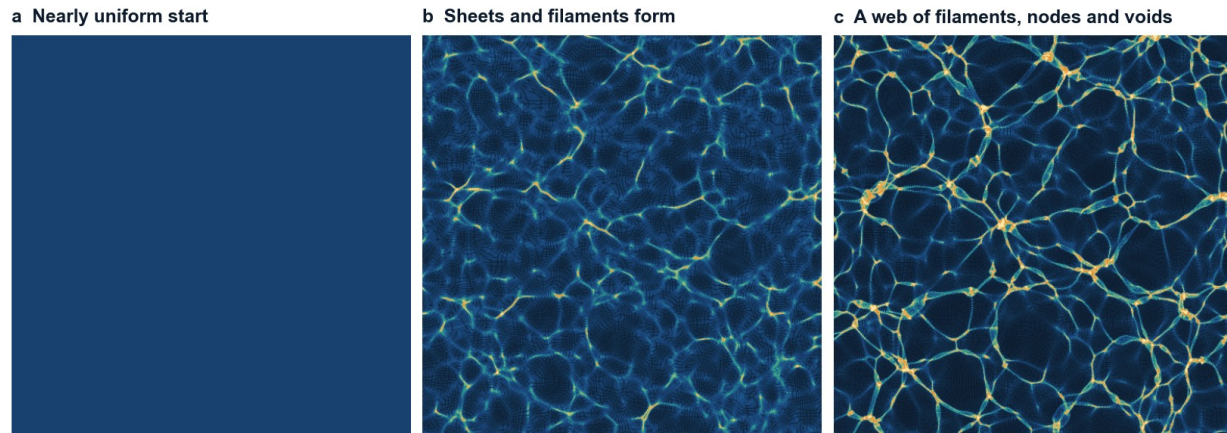


Figure 18. A cosmic web from gravitational instability. (a) Particles start almost uniformly distributed, with small random displacements. (b) As the displacements grow, matter collapses into sheets and filaments. (c) A web of filaments, dense nodes and empty voids emerges; in the final state about half the grid cells contain no particles.

Source: *H Heuristics* calculation using the Zel'dovich approximation in two dimensions: 262,144 particles displaced along the gradient of a Gaussian random potential with a power-law spectrum and small-scale cutoff, at three growth factors. Brightness is logarithmic in density.

Figure 18 reproduces the essential process with the Zel'dovich approximation. A quarter of a million particles were displaced along the gradient of a random potential, the displacement growing with time. From a nearly uniform start, sheets and filaments form where particle paths cross. In the final panel 49 per cent of the grid cells are empty and the densest 10 per cent of cells hold 54 per cent of the mass. The web is a product of *gravitational aggregation* acting on a *stochastic* initial field with a nearly scale-free spectrum, two of the seven mechanisms of Section 4, and its hierarchical character comes from small structures forming first and merging into larger ones.

7.3 Measuring the dimension of the galaxy distribution

The standard statistic for galaxy clustering is the two-point correlation function $\xi(r)$, the excess probability of finding a galaxy at distance r from another. Over a wide range of scales it is close to a power law, $\xi \propto r^{-\gamma}$ with $\gamma \approx 1.8$, reaching $\xi = 1$ at about $5 h^{-1}$ Mpc ([Davis and Peebles 1983](#)), where $h \approx 0.7$ and a megaparsec (Mpc) is about 3.3 million light years. A power-law correlation is the signature of a fractal: on scales where ξ is much larger than 1, the number of galaxies within distance r of a typical galaxy grows as $r^{3-\gamma}$, a correlation dimension near 1.2. As scale increases and ξ falls toward zero, the dimension rises toward 3, the value for a uniform distribution ([Jones et al. 2005](#)).

The scientific dispute was about how far the fractal regime extends. Pietronero and colleagues argued that the standard analysis, which normalizes counts by the mean density of a survey, hides fractality, and that galaxy counts are consistent with a dimension near 2 out to the largest scales then surveyed, with no sign of homogeneity ([Pietronero 1987](#); [Sylos Labini et al. 1998](#)). If true, the mean density of the universe would not be defined and standard cosmology would need revision.

The question was settled by surveys large enough to look beyond the clustering scale. Using luminous red galaxies from the Sloan Digital Sky Survey, Hogg and colleagues counted galaxies in spheres of increasing radius and found that counts grow as r^3 , the homogeneous value, on scales above about $70 h^{-1}$ Mpc ([Hogg et al. 2005](#)). The WiggleZ survey measured the correlation dimension directly as a function of scale and found it rising to within 1 per cent of 3 at similar scales ([Scrimgeour et al. 2012](#)), and the BOSS survey confirmed the transition with a much larger sample ([Ntelis et al. 2017](#)). Yadav and

colleagues showed that the fractal dimension itself is a useful way to define the scale of homogeneity (Yadav et al. 2010).

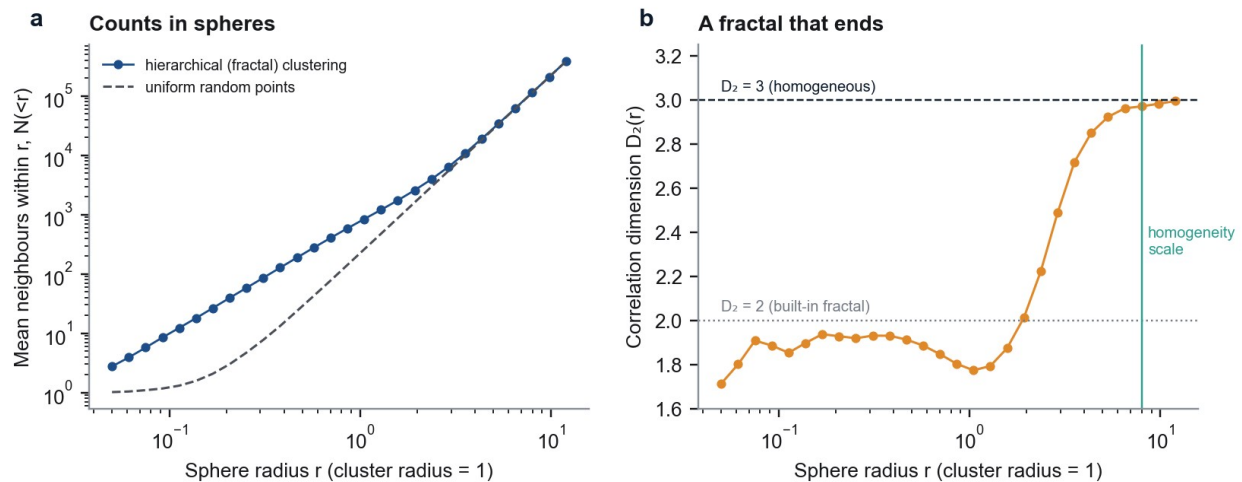


Figure 19. A fractal that ends. (a) Mean number of neighbours within radius r for a hierarchical (Soneira–Peebles) galaxy model with built-in dimension 2, scattered at random through a large volume, compared with uniform random points. (b) The local correlation dimension D_2 is close to 2 inside clusters and rises to 3, the homogeneous value, beyond them. Real surveys find an analogous transition near $70 \text{ h}^{-1} \text{ Mpc}$.

Source: H Heuristics calculation. 1,400 clusters of 1,024 points (5 levels, 4 subclusters per level, radius ratio 2) placed uniformly in a periodic box 30 cluster radii across; 1.43 million points; neighbour counts around 2,000 random points. Scales are in units of the cluster radius, not megaparsecs.

Figure 19 illustrates the logic with a model. The report built hierarchical clusters, each made of four subclusters, each of four smaller subclusters and so on for five levels, giving an exact built-in dimension of 2. It then scattered 1,400 such clusters at random through a large volume. Inside clusters the measured correlation dimension was about 1.91; beyond them it rose steeply, reaching 2.97 at about 8 cluster radii. The model is a cartoon, but it shows how a distribution can be genuinely fractal over a range and genuinely homogeneous beyond it, and how measuring the dimension as a function of scale, rather than fitting a single number, reveals both.

Table 8. Measurements bearing on the fractal dimension of the galaxy distribution

Study	Data	Finding
Davis and Peebles 1983	CfA redshift survey	Correlation function a power law, $\gamma \approx 1.8$, correlation length $\approx 5 \text{ h}^{-1} \text{ Mpc}$
Pietronero 1987 ; Sylos Labini et al. 1998	Several early catalogues	Argued for $D \approx 2$ with no observed homogeneity scale
Hogg et al. 2005	SDSS luminous red galaxies	Counts in spheres consistent with homogeneity above $\approx 70 \text{ h}^{-1} \text{ Mpc}$
Scrimgeour et al. 2012	WiggleZ, $z \approx 0.2\text{--}0.9$	D_2 rises to within 1% of 3 at roughly $70\text{--}80 \text{ h}^{-1} \text{ Mpc}$
Ntelis et al. 2017	BOSS DR12	Transition to homogeneity confirmed with a large sample

h is the dimensionless Hubble parameter, about 0.7, so $70 \text{ h}^{-1} \text{ Mpc}$ is roughly 100 Mpc . Homogeneity scales depend somewhat on the threshold chosen (for example, D_2 within 1% of 3).

7.4 Is the cosmic web multifractal?

The galaxy distribution is better described as multifractal than as a simple fractal: dense clusters and sparse filaments scale differently, and the generalized dimensions D_q vary with q on intermediate scales ([Jones et al. 2005](#)). This is what gravitational aggregation from a random initial field should produce, and it parallels the multifractality of turbulence and rainfall. It is also a reminder that the visual impression of 'the same pattern at every scale' is misleading: the cosmic web's filaments have a characteristic thickness, its voids a characteristic size of tens of megaparsecs, and its nodes a characteristic mass, all set by the physics of gravity and the initial spectrum.

Assessment. Matter in the universe is clustered in a scale-dependent, multifractal way from galactic scales to tens of megaparsecs (Grade B), and becomes statistically homogeneous above about 100 Mpc (Grade A). The universe is not fractal at all scales.

08

DOMAIN IV · CIVILIZATION



8. Civilization and Networks

8.1 The question

Research question. Are cities, infrastructure, markets and social networks fractal, and if so, is the resemblance to natural fractals more than metaphor?

Human systems are built by decisions rather than by physics alone, yet they show some of the most widely reported scaling laws in science. Urban street networks, settlement hierarchies, infrastructure, financial markets and online networks have all been described as fractal or scale-free. This domain is where the distinction between geometric resemblance and shared mechanism is hardest to draw, and where the critical turn of the past two decades has been most consequential.

8.2 Fractal cities

Batty and Longley showed in 1987 that the boundaries and built-up areas of cities have fractal dimensions, typically between about 1.2 for boundaries and 1.7 to 1.9 for built-up areas, depending on the city and the method ([Batty and Longley 1987](#)). Studies of urbanized areas in the United States found dimensions rising over time as cities filled in ([Shen 2002](#)). Two kinds of model reproduce such patterns. In diffusion-limited and dielectric-breakdown models, development accretes preferentially at the edges of existing development, producing branching footprints. In the correlated-percolation model of Makse, Havlin and Stanley, development probability decays with distance from the centre and is spatially correlated, reproducing both the fractal footprint and the size distribution of surrounding towns ([Makse et al. 1995](#)). Batty has argued that such models capture a real logic of urban growth, in which local decisions about access and adjacency aggregate into hierarchical, space-filling forms ([Batty 2008](#)).

The fractal description of cities is useful but limited. The scaling range of a city's footprint rarely exceeds two decades, from the block to the metropolitan region. Measured dimensions depend heavily on how 'built-up' is defined and on image resolution. And urban form is shaped by planning, topography and transport technology in ways that no single growth rule captures. Fractal cities are best graded C: a productive description with plausible mechanisms, not an established law.

8.3 Settlement hierarchies and Zipf's law

In many countries the size of a city is roughly inversely proportional to its rank: the second-largest city has about half the population of the largest, the third about a third, and so on. This is [Zipf's law](#), a power law with exponent close to 1. Gabaix showed that it follows if cities grow by random proportional amounts, each city's growth rate independent of its size, with a lower barrier that prevents cities from shrinking to nothing ([Gabaix 1999](#)). Figure 20b reproduces the result: 4,000 'cities' grew by random multiplicative

shocks for 4,000 periods, and the largest 400 settled on a rank–size slope of 0.906, close to Zipf's value of 1.

Zipf's law is often presented as evidence of fractal organization in settlement systems, echoing Walter Christaller's [central place theory](#), in which market towns are nested in a hexagonal hierarchy. The connection is real but partial. Zipf's law is a statement about a size distribution, not a geometry, and Cristelli, Batty and Pietronero showed that it holds only when the system of cities is defined coherently, for example as a whole nation, and fails for arbitrary subsets ([Cristelli et al. 2012](#)). The mechanism, proportional growth, is a statistical one that produces the same distribution in firm sizes, word frequencies and many other systems with no spatial structure at all.

Urban scaling laws make a related claim: across cities, quantities such as innovation and wages scale with population to a power slightly above 1, and infrastructure to a power slightly below 1 ([Bettencourt et al. 2007](#)). Bettencourt derived the exponents from a model in which city infrastructure is a space-filling, fractal network ([Bettencourt 2013](#)). This is an explicit attempt to give human settlements a fractal architecture in the sense of the West–Brown–Enquist model. It too has been contested: the exponents depend on how city boundaries are drawn ([Arcaute et al. 2015](#)), and some apparent non-linearity disappears under more careful statistics ([Leitão et al. 2016](#)).

8.4 Scale-free networks

In 1999 Barabási and Albert showed that a network that grows by adding nodes which link preferentially to already well-connected nodes develops a power-law degree distribution: a few hubs with enormous numbers of links and a long tail of nodes with few ([Barabási and Albert 1999](#)). Such networks have no characteristic number of links and were dubbed scale-free. They were soon reported everywhere, from the World Wide Web and the Internet's router topology ([Faloutsos et al. 1999](#)) to protein interactions and citation networks ([Albert and Barabási 2002](#)). Song, Havlin and Makse showed that some complex networks are also self-similar in a stricter sense: covering them with boxes of increasing size reveals a fractal dimension ([Song et al. 2005](#)), and later work unfolded real networks across scales by geometric renormalization ([García-Pérez et al. 2018](#)).

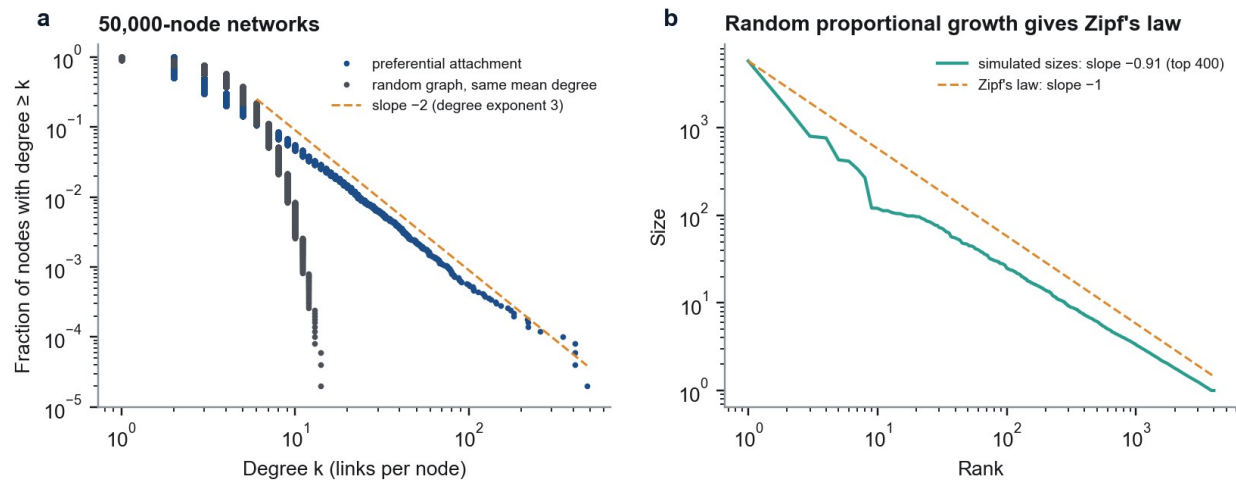


Figure 20. Proportional growth produces heavy tails. (a) Degree distributions of a 50,000-node network grown by preferential attachment and a random network with the same mean degree. The first has hubs with hundreds of links; the second has none above 14. (b) Rank–size plot for 4,000 'cities' grown by random proportional shocks with a lower barrier, which settle near Zipf's law.

Source: H Heuristics calculation. Barabási–Albert model with two links per new node; Poisson random graph with equal mean degree. City model: multiplicative lognormal shocks (mean -0.2% , s.d. 6% per period) for 4,000 periods with a reflecting floor; slope fitted to the top 400 ranks.

Figure 20a shows the contrast. In the report's preferential-attachment network the best-connected node had 480 links; in a random network of the same size and mean degree the maximum was 14. But whether real networks are scale-free is now disputed. Broido and Clauset applied rigorous tests to nearly a thousand networks from biology, technology, society and transport and found that strongly scale-free structure is rare: for most networks a lognormal or other distribution fits as well as or better than a power law (Broido and Clauset 2019). Others replied that heavy-tailed, hub-dominated structure, whatever its precise form, is common and consequential (Holme 2019). Both points can be true. The lesson for this report is that 'scale-free' in the strict mathematical sense, a pure power law across many decades, is a stronger claim than the data usually support.

8.5 Markets

Mandelbrot's first fractal discovery concerned prices. Returns on cotton, and later on stocks and currencies, have heavy tails, and their statistical properties look similar over a range of time horizons (Mandelbrot 1963). Volatility clusters, so that turbulent periods follow turbulent periods, and the magnitude of returns shows long memory. These features are well established and have practical consequences: models that assume normally distributed returns badly underestimate the probability of crashes. Mandelbrot and colleagues later proposed multifractal models of asset returns, analogous to turbulent cascades, and multifractal analysis methods have been widely applied to financial series (Kantelhardt et al. 2002). Whether markets are multifractal in a deep sense, or merely heavy-tailed with volatility clustering for reasons of trader behaviour and institutional structure, is unresolved (Grade B for heavy tails and scaling over horizons; Grade C for multifractal cascades as a mechanism).

8.6 Art, architecture and perception

Fractal patterns appear in human design across cultures: in the nested domes and spires of Hindu temples, in Gothic tracery, in Islamic geometric ornament and, as Ron Eglash documented, in the

recursive layouts of many African settlements and textiles ([Eg lash](#)). In 1999 Richard Taylor and colleagues reported that Jackson Pollock's drip paintings are fractal, with dimensions that increased over his career, and proposed fractal analysis as a tool for authentication ([Taylor et al. 1999](#)). Jones-Smith and Mathur challenged the analysis, arguing that the scaling ranges were too short to support the claim and that simple non-Pollock drawings could pass the test ([Jones-Smith and Mathur 2006](#)). The dispute is a textbook case of the problems discussed in Section 11.

A more robust finding concerns perception. Experiments find that people tend to prefer fractal patterns of intermediate complexity, with dimensions roughly between 1.3 and 1.5, across natural scenes, artworks and computer-generated images ([Spehar et al. 2003](#)), and that viewing such patterns is associated with measurable physiological responses ([Taylor et al. 2011](#)). A plausible explanation is that human vision evolved in environments dominated by statistically fractal scenes. If so, part of the apparent ubiquity of fractals in human design is a fact about the observer.

8.7 Assessment

Table 9. Fractal and scaling claims in human systems, graded

Claim	Evidence	Mechanism	Grade
City footprints are fractal	$D \approx 1.2\text{--}1.9$ over one to two decades; definition-dependent	Edge-biased aggregation; correlated percolation	C
City sizes follow Zipf's law	Holds for coherent national systems, exponent near 1	Random proportional growth	B
Urban indicators scale non-linearly	Exponents sensitive to boundaries and methods	Fractal infrastructure networks (proposed)	C
Many networks are heavy-tailed with hubs	Widely observed	Preferential attachment, among others	B
Most real networks are strictly scale-free	Rare under rigorous tests	Disputed	C
Asset returns are heavy-tailed and scale over horizons	Replicated across markets and decades	Behavioural and institutional (not settled)	B
Pollock's paintings are fractal and datable by D	Short scaling ranges; contested	Not applicable	C
People prefer mid-range fractal dimensions	Replicated perceptual experiments	Evolved visual processing (proposed)	B

Human systems mix physical, statistical and cultural processes, and their scaling ranges are usually short. Grades are correspondingly lower than for Earth systems.

Civilization supports H1 only partially. Heavy-tailed size distributions and hub-dominated networks are common, but strict fractal geometry over two or more decades is rare, and many celebrated claims have weakened under testing. H2 fares better: proportional growth and edge-biased aggregation reproduce the main regularities. But because proportional growth is a statistical mechanism that operates in many systems with no shared physics, the regularities it explains say less about a shared architecture than a casual reading suggests.

09

SIMILAR FORMS, DIFFERENT CAUSES



9. Similar Forms, Different Causes

9.1 The question

Research question. When does a shared fractal description reveal a genuinely shared principle, and when does it merely describe different systems in comparable language?

This is the pivotal question of the report. Sections 5 to 8 found fractal-like structure in all four domains. A vascular network and a river basin both branch; a cloud and a coastline both have rough boundaries; a galaxy survey and an aggregate both show power-law clustering. The Fractal Architecture Hypothesis is interesting only if some of these resemblances point to a common cause. This section distinguishes geometric resemblance from shared generative mechanism and proposes a way of telling them apart.

9.2 Resemblance and mechanism

Table 10 sets out the recurring patterns of the previous sections, the mechanisms that can produce each, and why the pattern matters. The table's key column is the last: whether the pattern, when found in two systems, implies a shared principle.

Table 10. Recurring fractal patterns, possible mechanisms and what a shared pattern implies

Pattern	Possible mechanisms	Why it matters	Does a shared pattern imply a shared principle?
Branching	Transport optimization; recursive growth; diffusion-limited growth; erosion	Reaches many locations through connected pathways at low cost	Only if branching ratios, exponents and cutoffs also match. Shared transport optimization is a real common principle
Repeated geometric forms	Recursive growth; iteration of a developmental rule	Produces complexity from simple rules and little information	Rarely: the rule is usually system-specific
Scale-dependent clustering	Gravitational or diffusive aggregation; cascades	Creates organization across spatial scales	Yes within a universality class; otherwise only statistically
Irregular boundaries	Erosion and competing forces; thresholding of rough fields; growth instabilities	Produces complex interfaces and large contact areas	Sometimes: boundaries of rough fields share universal exponents
Hierarchical networks and heavy tails	Proportional growth; preferential attachment; optimization	Connects local and large-scale structure; creates hubs	Weakly: proportional growth gives similar statistics in unrelated systems

Pattern	Possible mechanisms	Why it matters	Does a shared pattern imply a shared principle?
Power-law event sizes	Criticality, self-organized or tuned; cascades	Makes extreme events far more likely than in normal statistics	Yes if exponents match a universality class; otherwise no

Expanded from the pattern–mechanism table in the series' research brief. 'Shared principle' means a generative mechanism or governing equation, not merely a shared mathematical description.

9.3 Universality: the strongest form of fractal unity

In statistical physics, [universality](#) is the discovery that systems with completely different microscopic make-up behave identically near a critical point, sharing exact exponents, because the details wash out at large scales. Water near its critical point and a magnet near its Curie temperature fall in the same universality class. Fractal growth has its own universality classes, and they provide the clearest cases in which fractal similarity reveals a shared principle.

The best example is diffusion-limited aggregation. In the Witten–Sander model, random walkers stick to a growing cluster. In electrodeposition, metal ions diffuse to a cathode and are deposited. In dielectric breakdown, an electrical discharge advances where the electric potential gradient is steepest. In viscous fingering, a low-viscosity fluid pushed into a high-viscosity one advances where the pressure gradient is steepest. These are different materials and forces, but in each case the growth rate at the interface is governed by the gradient of a field that satisfies Laplace's equation. Because they share that governing equation, they share a fractal dimension, close to 1.71 in two dimensions. Experiments confirm it in zinc deposits ([Matsushita et al. 1984](#)), discharges ([Niemeyer et al. 1984](#)) and porous media ([Måløy et al. 1985](#)).

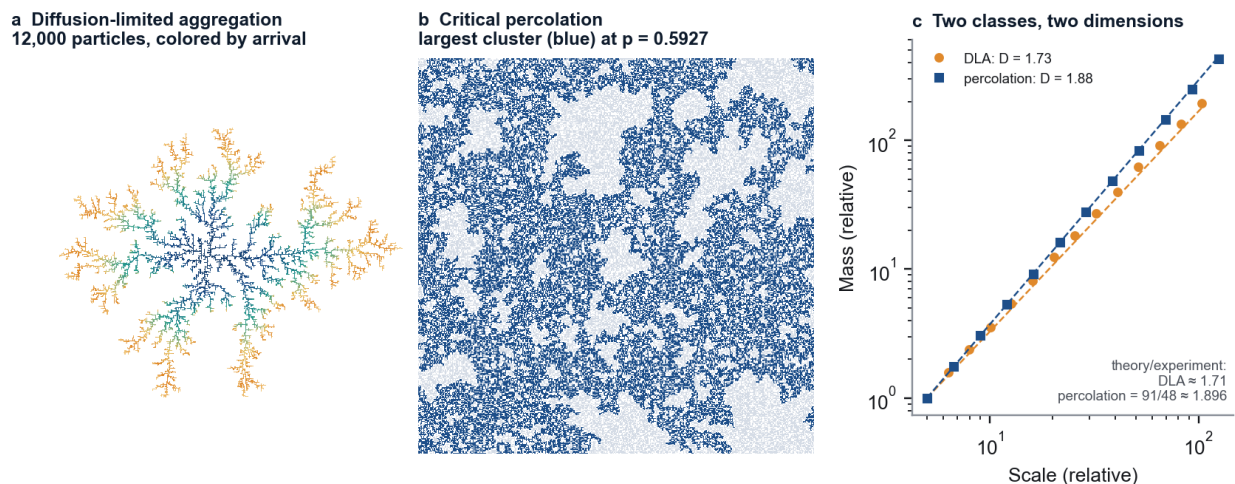


Figure 21. Two universality classes, two dimensions. (a) A diffusion-limited aggregate of 12,000 particles, coloured from early (blue) to late (orange) arrival. (b) The largest cluster at the percolation threshold. (c) Mass against scale: the aggregate gives $D \approx 1.73$ and the percolation cluster $D \approx 1.88$, close to the theoretical and experimental values of 1.71 and $91/48 \approx 1.896$.

Source: H Heuristics calculation. On-lattice DLA on a $1,001 \times 1,001$ grid, mass–radius dimension fitted from radius 5 to about half the cluster radius. Site percolation on a $1,024 \times 1,024$ lattice at $p = 0.592746$; correlation-integral dimension fitted over 4–100 sites.

Figure 21 grows a diffusion-limited aggregate and a cluster at the [percolation](#) threshold, the point at which randomly occupied sites first form a connected path across a lattice. The aggregate's mass grew with radius to the power 1.733; the percolation cluster's to the power 1.882, against an exact theoretical value

of $91/48 \approx 1.896$. These numbers are not fitted to any natural system. They are properties of mathematical classes, and any physical system that belongs to a class must reproduce them. Percolation exponents recur in porous media, the spread of fires and epidemics on networks, and the electrical conductivity of composites; percolation-hull exponents recur in coastline models (Section 6.2). Kolmogorov's $-5/3$ law recurs across every turbulent fluid.

Key finding. Universality is the rung of the ladder at which fractal geometry becomes evidence of a shared principle. Its signature is not a shared appearance but a shared number, derived from a shared equation and reproduced across materials.

9.4 The vascular network and the river basin

The research brief for this report singled out one comparison: a vascular network and a river basin both branch, yet they are not generated by identical processes. Table 11 compares them on the five tests of Section 4.

Table 11. A vascular tree and a river network compared on the five tests

Test	Vascular tree	River network	Verdict
1. Range	~3 decades of radius, ~20–30 generations	Several decades of drainage area	Both pass
2. Statistics	Branching ratios consistent across organs and species	Horton and Hack laws replicated worldwide	Both pass
3. Numbers	Radius ratio $2^{-1/3}$ (Murray) to $2^{-1/2}$; 3D volume-filling	Hack exponent ≈ 0.5 – 0.6 ; network embedded in a 2D surface	Different exponents
4. Mechanism	Developmental program tuned by selection to minimize pumping and upkeep	Erosion feedback approximating minimum energy dissipation	Different processes, related optimum
5. Texture	Distributes from one source to many sinks; loops in capillary beds	Collects from many sources to one outlet; strictly tree-like	Differ

Direction of flow, embedding dimension and generating process all differ, yet both approximate solutions to the same class of problem: efficient transport between a point and a region.

The verdict is instructive. The two systems fail the test of shared numbers and shared texture, and their generating processes are different: one is a genetic and developmental program refined by natural selection, the other the self-organization of erosion. Yet they are not unrelated. Banavar, Maritan and Rinaldo showed that *any* efficient network that transports material between a point and every part of a D -dimensional region obeys a general scaling relation between the volume of material in transit and the size of the region served ([Banavar et al. 1999](#)). In three dimensions it yields metabolic scaling to the $3/4$ power without requiring the network to be fractal; in two dimensions it constrains river basins. The deep commonality between vessels and rivers is therefore not their fractal geometry as such but the optimization problem they both approximately solve. Fractal branching is one efficient solution; the principle is efficiency of transport under geometric constraints.

This is the most important conceptual result of the section. In the best-understood cases, fractal geometry is a *symptom* of a shared principle, not the principle itself. The principle is an equation, an optimization problem or a statistical law. Fractal form appears because it is how that principle is realized in space.

9.5 Same dimension, different origin

If a shared dimension implied a shared mechanism, two systems with equal dimension would be generated alike. Figure 22 tests this directly. The report took its diffusion-limited aggregate and measured its correlation dimension as 1.64. It then built a completely different object, a Cantor dust made by recursively keeping the four corners of a square and discarding the rest, with the construction tuned until its measured dimension matched: 1.644. One object grew by random accretion; the other was carved by deterministic deletion. A single number cannot tell them apart.

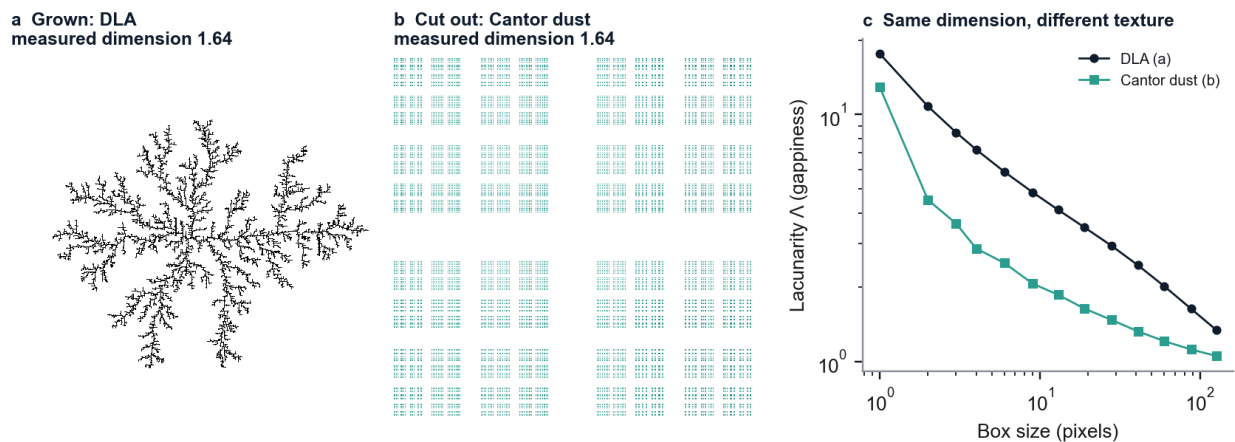


Figure 22. Same dimension, different origin. (a) A diffusion-limited aggregate grown by random walkers. (b) A Cantor dust cut out by a deterministic rule, tuned to the same measured dimension. (c) Lacunarity, a measure of gappiness, differs between them by about a factor of two at intermediate scales, distinguishing objects that dimension alone cannot.

Source: H Heuristics calculation. Both images 460×460 pixels; dimension from the correlation integral over 3–60 pixels; gliding-box lacunarity $\Delta(r) = \langle M^2 \rangle / \langle M \rangle^2$ for box sizes 1–128 pixels.

A second measure can. [Lacunarity](#) quantifies how gappy a structure is: how unevenly its mass is spread among boxes of a given size. At a box size of 16 pixels the dust's lacunarity was 0.45 times that of the aggregate, and the two curves have different shapes. Lacunarity, multifractal spectra, branching ratios and topology are what Test 5 calls texture, and they are essential for any claim of shared architecture. Dimension is a necessary condition for similarity, not a sufficient one.

9.6 The brain and the cosmic web

Few comparisons capture the popular imagination like the resemblance between images of neurons and of the cosmic web. In 2020 Vazza and Feletti compared the two quantitatively, using networks built by connecting nearby neurons in brain tissue and nearby galaxies or dark-matter haloes in a simulation, and reported a striking similarity in their network statistics and in the power spectrum of their density fluctuations over certain ranges ([Vazza and Feletti 2020](#)). They suggested that both systems may be shaped by similar principles of network self-organization.

Applying the five tests clarifies what such a result can and cannot establish. The comparison passes a version of Test 2, since quantitative statistics were computed for both, and partly Test 3, since some numbers were similar. It fails Test 4: the cosmic web is built by gravity acting on dark matter over billions of years from a nearly scale-invariant initial spectrum, while neural tissue is built by gene expression, cell migration and activity-dependent wiring over months. The authors themselves noted that their neuronal network was a proximity approximation, not real connectivity. And the systems differ in scale by about 27

orders of magnitude, so no physical process links them. A shared principle, if there is one, would have to be abstract, such as a general tendency of networks formed by local aggregation to have similar statistics. That is a reasonable research hypothesis, but on present evidence the resemblance belongs on rung 1 or 2 of the ladder, not rung 4 (Grade D for a shared principle).

9.7 A map of mechanisms

Figure 23 summarizes the report's assessment of which mechanisms operate in which systems. Three features stand out. First, most systems are dominated by one or two mechanisms, which makes the mechanism, not the domain, the natural unit of comparison. Second, mechanisms cross domains: aggregation links electrodeposits, galaxy clustering and urban footprints; cascades link turbulence, rainfall and financial volatility; transport optimization links vessels, airways, plants and rivers. Third, the systems with the highest evidence grades are those with a single, well-modelled mechanism.

	Transport optimization	Recursive growth	Aggregation	Criticality	Cascade / stochastic	Erosion / competition	Proportional growth	Grade
Arterial and venous trees	primary mechanism	important contributor	minor or disputed role	not involved	not involved	not involved	not involved	B
Bronchial tree (lung)	primary mechanism	important contributor	minor or disputed role	not involved	not involved	not involved	not involved	B
Plant branching and xylem	important contributor	primary mechanism	minor or disputed role	not involved	not involved	not involved	not involved	B
Romanesco inflorescence	not involved	primary mechanism	minor or disputed role	not involved	not involved	not involved	not involved	B
Neuronal arbors	important contributor	important contributor	minor or disputed role	not involved	important contributor	not involved	not involved	C
Chromatin folding	not involved	not involved	important contributor	not involved	important contributor	not involved	not involved	C
River networks	primary mechanism	not involved	not involved	important contributor	important contributor	primary mechanism	not involved	B
Coastlines	not involved	not involved	not involved	important contributor	important contributor	primary mechanism	not involved	B
Clouds and rain areas	not involved	not involved	important contributor	not involved	primary mechanism	not involved	not involved	B
Turbulence	not involved	not involved	not involved	not involved	primary mechanism	not involved	not involved	A
Earthquakes and faults	not involved	not involved	not involved	important contributor	important contributor	important contributor	not involved	B
Electrodeposits, lightning	not involved	not involved	primary mechanism	important contributor	important contributor	not involved	not involved	A
Galaxy distribution (< ~100 Mpc)	not involved	not involved	primary mechanism	not involved	important contributor	not involved	not involved	B
Urban footprints	important contributor	not involved	important contributor	not involved	important contributor	not involved	important contributor	C
City-size distribution	not involved	not involved	not involved	not involved	important contributor	not involved	primary mechanism	B
Internet and web graphs	important contributor	not involved	not involved	not involved	important contributor	not involved	primary mechanism	C
Financial returns	not involved	not involved	not involved	important contributor	primary mechanism	not involved	important contributor	C

primary mechanism important contributor minor or disputed role not involved

Grade: evidence that the system is fractal over a substantial range (A strongest, see Table 1).

Figure 23. Which mechanisms generate which fractal systems. Darker cells mark the more important mechanisms; the right-hand column gives the evidence grade for fractal structure over a substantial range. Mechanisms cross domains, and the best-established fractals have one dominant, well-modelled mechanism.

Source: H Heuristics assessment based on the literature reviewed in Sections 5–8. Cell shading is a qualitative judgement, not a measurement.

9.8 Four kinds of fractal unity

The evidence suggests that 'shared fractal architecture' can mean four different things, of decreasing depth (Table 12).

Table 12. Four kinds of fractal unity

Kind	What is shared	Example	Depth
Universal	A governing equation and therefore exact exponents	DLA in deposits, discharges and fingering; percolation clusters; turbulence	Deep: a shared law
Functional	An optimization problem, approximately solved	Vascular trees, airways, plant xylem, river networks	Substantial: a shared constraint
Statistical	A generic random process or limit theorem	Zipf's law from proportional growth in cities, firms and words; Horton ratios	Thin: a shared statistic
Descriptive	Only the mathematical language	Neurons and the cosmic web; fractal art and coastlines	None demonstrated

The four kinds correspond roughly to rungs 4, 3, 2 and 0–1 of the ladder in Figure 10. Only the first two support the foundational level of the Fractal Architecture Hypothesis.

The strongest research contribution of the fractal program, in this light, is not to find more fractals. It is to determine, for each resemblance, which of the four kinds of unity it represents.

10

WHY FRACTAL STRUCTURE PERSISTS



10. Why Fractal Structure Persists

10.1 The question

Research question. Once generated, why does fractal structure persist? Is it selected for function, maintained by physical optimization, held by the dynamics of criticality, or simply statistically inevitable?

In the Architecture of Creation framework, forms persist because they pass filters: they are stable, efficient, reproducible or selected. The previous section asked what generates fractal structure. This section asks what keeps it. The answer differs between living, physical and statistical systems, and recognizing four distinct routes to persistence clarifies why fractals are common without implying that nature prefers them.

10.2 Function: why branching pays

The most direct reason fractal structure persists in organisms is that it does a job cheaply. Consider the problem of supplying N points spread evenly through a region, such as cells needing oxygen or leaves needing water. One solution is a separate pipe from the source to every point. Another is a branching hierarchy. Figure 24 computes the total pipe length needed by each. For a thousand points the hierarchy needs 8.4 times less pipe; for a million points, 261 times less. The advantage grows as the square root of N , so for the billions of cells in a large organism it is overwhelming.

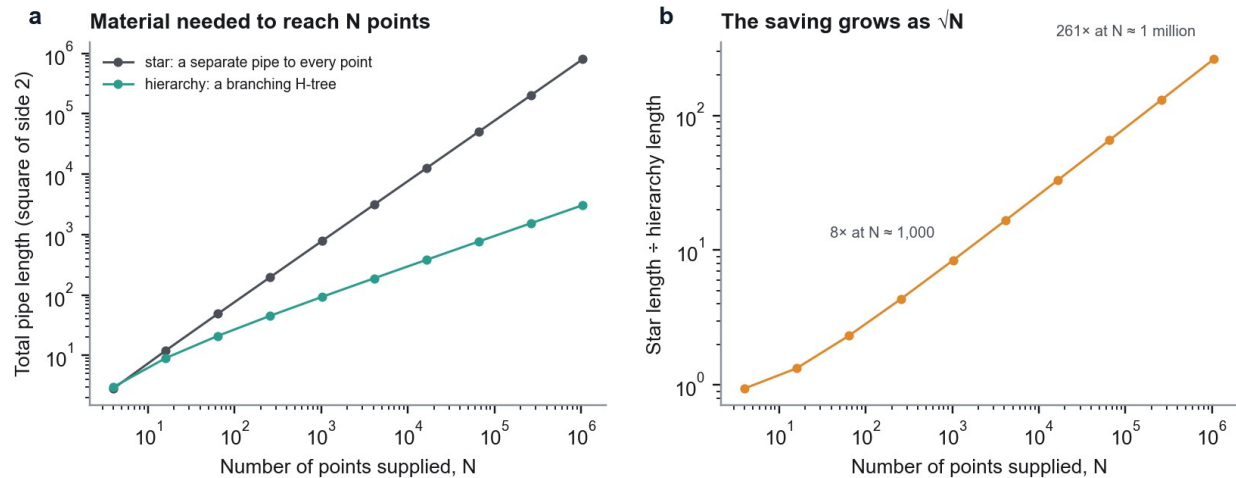


Figure 24. Why branching pays. (a) Total pipe length needed to supply N evenly spaced points in a square, by a star of separate pipes and by a branching H-tree. (b) The ratio grows as the square root of N : about 8 times at a thousand points and about 260 times at a million.

Source: H Heuristics calculation. Points on a regular grid filling a square of side 2; star length is the sum of distances from the centre; H-tree length summed over levels with segment halving at alternate levels.

Function explains other features of biological fractals. Surface area for exchange, in lungs, gills, intestinal villi, root systems and the folded cortex, can be made very large within a small volume by repeated folding or branching, because a fractal surface's area grows faster than its enclosing volume as resolution increases. Robustness benefits too: a hierarchy with many parallel paths at its fine levels tolerates local damage, and the lung's deliberate departure from the strict optimum (Section 5.3) shows selection trading efficiency for safety. Finally, a recursive rule is informationally cheap: a genome need not specify every vessel, only a branching rule and when to stop.

10.3 Optimization without selection

Non-living systems can also arrive at near-optimal fractal structures, by physical feedback rather than selection. River networks are the clearest case. Erosion is fastest where flow is concentrated, which deepens channels and captures more flow, a positive feedback that organizes the landscape. Networks that minimize total energy dissipation reproduce the observed river statistics, and real networks evolve toward such configurations, though they become trapped in local optima determined by their history (Rinaldo et al. 2014). Coastlines settle into fractal shapes that minimize erosion by dissipating wave energy (Sapoval et al. 2004). In both cases fractal form persists because it is a dynamically stable state: departures from it are corrected by the same processes that created it.

10.4 Criticality as an attractor

Some fractal structure persists because a system's dynamics drive it to a critical state and hold it there. In equilibrium physics, critical fractals such as percolation clusters or the fluctuations of a fluid at its critical point require fine tuning: move the temperature or density slightly and the fractal disappears. Self-organized criticality proposes that slowly driven, threshold-dominated systems tune themselves. The sandpile of Figure 17 reached its critical state from any starting configuration and stayed there. The concept has been applied to earthquakes, forest fires, solar flares and neuronal avalanches. Its generality is debated: many systems that produce power laws do so for other reasons, and some proposed

examples require hidden tuning. But where it applies, it explains persistence elegantly: fractal statistics are the attractor of the dynamics.

10.5 Statistical inevitability

The fourth route is the least glamorous and the most often overlooked. Some fractal and scaling patterns persist because they are what random processes generically produce, much as the bell curve persists because averages of many independent effects converge to it. Random proportional growth converges to Zipf-like distributions (Figure 20b). Almost any random branching network satisfies Horton's laws (Kirchner 1993). Sums of heavy-tailed variables converge to stable distributions with power-law tails. Such patterns are robust precisely because they do not depend on details, which also means they carry little information about the system that shows them. Their persistence is real, but it is not evidence of design or optimization.

10.6 Four routes compared

Table 13. Four routes by which fractal structure persists

Route	What maintains the structure	Examples	Testable prediction
Function and selection	Natural selection favouring cheap, robust, space-filling designs	Vessels, airways, xylem, neurons, cortex	Departures from fractal ratios track costs and risks (e.g. the lung's safety margin)
Physical optimization	Feedback that drives a system toward a minimum of dissipation or stress	River networks, coastlines, some fracture patterns	Systems relax back to fractal statistics after perturbation
Criticality	Dynamics that drive a system to, and hold it at, a critical point	Sandpile models; possibly earthquakes and neuronal avalanches	Exponents match a universality class; cutoffs scale with system size
Statistical inevitability	Generic limit behaviour of random processes	Zipf's law, Horton ratios, heavy-tailed sums	Pattern appears in randomized null models with no optimization

A single system can persist by more than one route. River networks, for instance, combine physical optimization with the statistical inevitability of Horton's laws.

The four routes map onto the series framework. Function and physical optimization are *filters*: they select among possible forms. Criticality is a *generator* that is its own filter. Statistical inevitability is a *constraint*: it defines what most random outcomes look like. Fractal structure is common because all four routes lead to it, not because any one of them is universal.

11

WHERE FRACTALITY BREAKS DOWN



11. Where Fractality Breaks Down

11.1 The question

Research question. Where do fractal descriptions fail, and how can a genuine fractal be distinguished from an apparent one?

A fair test of the Fractal Architecture Hypothesis must look as hard for failures as for successes. Fractal descriptions break down in two ways. Real fractals end: every physical fractal has scales at which its self-similarity stops, and those cutoffs are as informative as the scaling between them. And apparent fractals deceive: objects and data that are not fractal can produce convincing fractal signatures. This section examines both.

11.2 Every physical fractal ends

The inner and outer cutoffs of a fractal are set by physics. A coastline's roughness ends at the grain of its sediment and at the size of its continent. The bronchial tree ends at the alveolus, whose size is set by the diffusion distance of oxygen, and at the trachea, whose size is set by the body. Galaxy clustering ends, at the upper scale, where the universe becomes homogeneous (Figure 19), because the initial fluctuations had too little power on very large scales to build structure there in the time available. Turbulence ends at the large scale where energy is injected and at the small Kolmogorov scale where viscosity takes over. The sandpile's avalanches end at the size of the grid (Figure 17).

Cutoffs carry information that the scaling range does not. The ratio of a fractal's outer to inner cutoff measures how many levels the generating process can sustain. The location of a cutoff often identifies the competing process that ends the fractal regime: viscosity in turbulence, diffusion in the lung, gravity's finite age in cosmology. A claim that a system is fractal 'at all scales' is therefore not merely unproven; it is a claim that no competing process ever intervenes, which is false for every physical system examined in this report.

11.3 Limited scaling ranges

In 1998 Avnir, Biham, Lidar and Malcai examined experimental fractal dimensions reported in the *Physical Review* journals over several years and found that the scaling ranges supporting them were typically short, centring on about 1.3 decades and seldom exceeding two ([Avnir et al. 1998](#)). They asked whether many reported fractals were artefacts of fitting straight lines to short stretches of curved log-log plots.

Figure 25 shows why the question matters. The report scattered two kinds of object that are definitely not fractal: random straight sticks, each of dimension 1, and random solid discs, each of dimension 2, both

with sizes spread over a wide range, as particle sizes often are in real materials. Box counting on the sticks shows the local slope rising from 1 to 2 as boxes grow larger than the sticks. Box counting on the discs shows a slope that falls from 2 and then recovers, passing through intermediate values. Within the disc curve the report searched for a 1.3-decade window in which a straight line fitted almost perfectly. It found one, between 2 and 45 pixels, giving a 'fractal dimension' of 1.5 with $R^2 = 0.9955$. By the standards of many published studies, that would be reported as a fractal.

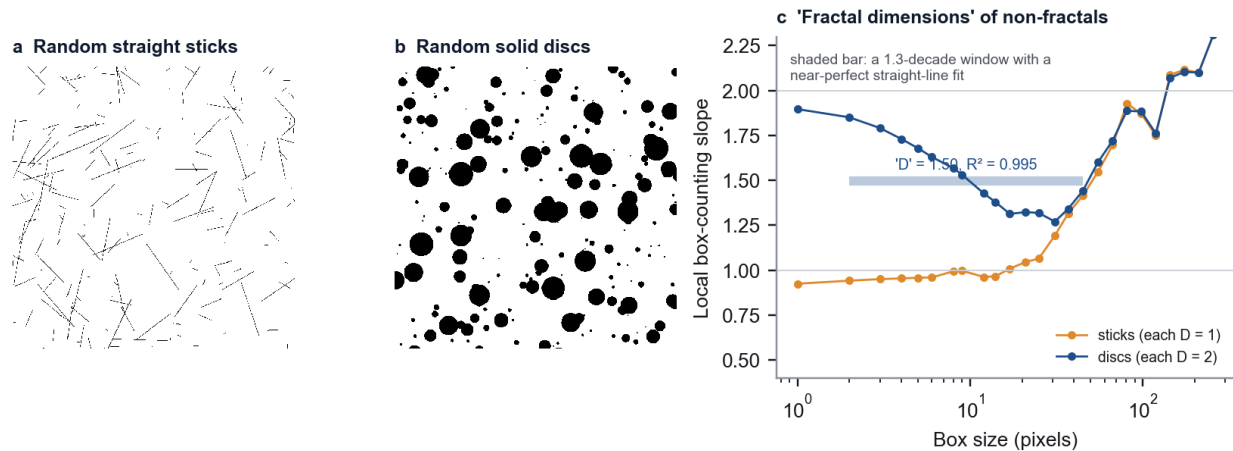


Figure 25. Non-fractals that pass as fractals. (a) Random straight sticks and (b) random solid discs, both with log-uniform size distributions. (c) Local box-counting slope against box size. Neither object is fractal, but the slope wanders between integer values, and a 1.3-decade window of the disc curve gives a near-perfect straight-line fit with a non-integer 'dimension'.

Source: H Heuristics calculation. 2,048 × 2,048 pixel images; 1,500 sticks of length 4–160 pixels; 1,800 discs of radius 1.5–30 pixels; local slopes from the box-counting curve for box sizes 1–256 pixels.

Key point. A straight line over one decade on a log-log plot is weak evidence of fractality. Two or more decades, the local slope plotted against scale, and a physical reason for the cutoffs are the minimum for a credible claim.

11.4 Power laws that are not

Fractal structure and power-law distributions are closely linked: a fractal's mass, number of parts or event sizes typically follow power laws. Testing a power law is therefore part of testing a fractal. The standard practice of plotting a histogram or cumulative distribution on log-log axes and fitting a line is unreliable. Clauset, Shalizi and Newman set out a better procedure: estimate the power-law exponent by maximum likelihood above a cutoff chosen objectively, test the fit's plausibility, and compare it against alternative distributions using a likelihood-ratio test (Clauset et al. 2009). Applied to two dozen published data sets, the procedure found several claimed power laws inconsistent with the data and many others indistinguishable from lognormals.

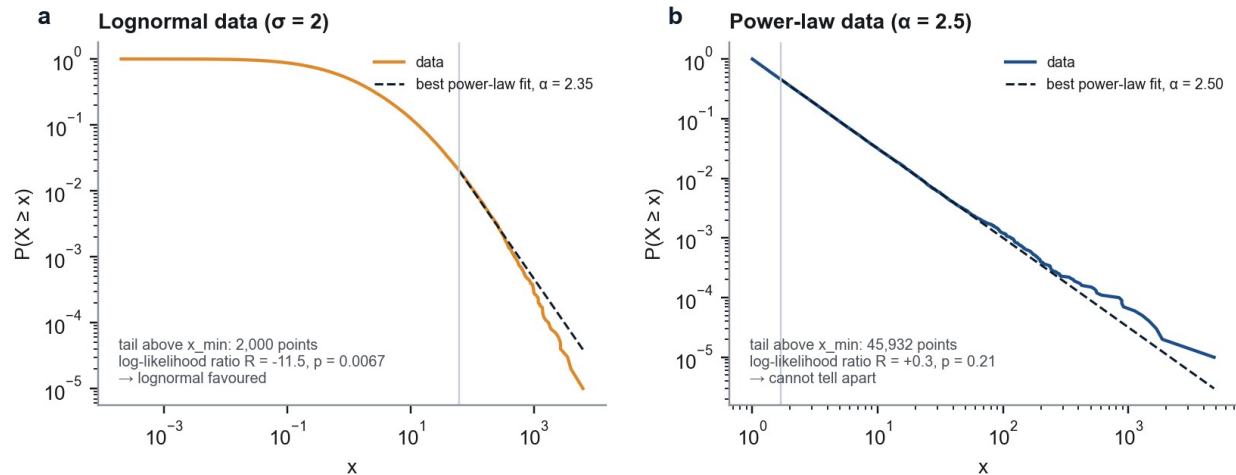


Figure 26. A lognormal can look like a power law. (a) The tail of 100,000 lognormal values is close to straight on log-log axes, and a power law fitted above an objectively chosen cutoff looks plausible; a likelihood-ratio test favours the lognormal. (b) For true power-law data the fit is excellent, yet a flexible lognormal fits the tail almost as well, and the test cannot exclude it.

Source: H Heuristics calculation. Lognormal ($\sigma = 2$) and Pareto (density exponent 2.5) samples of 100,000; power law fitted by maximum likelihood with $x_{\min}$ chosen to minimize the Kolmogorov–Smirnov distance; truncated lognormal fitted by maximum likelihood on the same tail; Vuong likelihood-ratio test.

Figure 26 applies the procedure. For lognormal data, the tail above the chosen cutoff was 2,000 values, a power law with exponent 2.35 looked like a reasonable fit, and only the likelihood-ratio test ($R = -11.5$, $p = 0.0067$) revealed that the lognormal was better. For genuine power-law data, the test returned $R = 0.3$ with $p = 0.21$: the power law was not rejected, but neither was the lognormal. Distinguishing the two requires very large samples spanning many decades, which most natural and social data sets lack. The appropriate conclusion in such cases is 'heavy-tailed', not 'fractal' or 'scale-free'.

11.5 Other failure modes

Table 14. Ways in which fractal claims fail

Failure mode	What happens	Example	Safeguard
Short scaling range	A curved relation looks straight over a decade	Many published experimental dimensions (Avnir et al.)	Require ≥ 2 decades; plot local slopes
Wrong distribution	Lognormal or truncated distributions mistaken for power laws	Many 'scale-free' networks (Broido and Clauset)	Maximum-likelihood fits and model comparison
Crossover between regimes	Two different scaling regimes averaged into one exponent	Topography with process-dependent dimension	Measure dimension as a function of scale
Self-affinity treated as self-similarity	Dimension depends on method and direction	Profiles, time series, fracture surfaces	Report Hurst exponents and anisotropy
Finite-sample bias	Sparse sampling underestimates dimension	Random walk measured at 1.74 instead of 2	Convergence tests with sample size
Multifractality ignored	One dimension used for a spectrum	Rainfall, turbulence, galaxy clustering	Estimate generalized dimensions
Resolution artefacts	Pixel size or ruler choice sets the apparent range	Fractal analysis of paintings and city images	Vary resolution independently of scale

Failure mode	What happens	Example	Safeguard
Visual pattern-seeking	Resemblance taken as evidence	Neurons and the cosmic web; galaxies and brains	Apply the five tests of Section 4

Several of these failure modes can combine. A short scaling range measured at a single resolution with a least-squares fit to a histogram is common in the literature and can produce a ‘fractal dimension’ from almost any data.

11.6 Multiscale, not fractal

Many of the systems in this report are better described as *multiscale* than as fractal. They have structure at many scales, but the structure changes with scale: a different exponent in each regime, characteristic sizes at the boundaries between regimes, and anisotropy that depends on direction. The universe has stars, galaxies, groups, clusters, filaments and voids, each with characteristic sizes. A tree has a trunk, boughs, twigs and leaves, which are not miniature trees. A city has buildings, blocks, neighbourhoods and districts. Fractal geometry captures part of this organization, the part over which a single rule repeats, and its great virtue is that it measures that part exactly. But treating multiscale organization as if it were scale-free throws away the information in the characteristic scales, which is often where the physics is.

Assessment. Fractality breaks down at cutoffs set by competing physics, and apparent fractality often reflects short scaling ranges, poorly chosen distributions or visual pattern-seeking (Grade A for the existence of these problems). The general description of reality that survives is multiscale structure, of which strict fractals are an idealized and limited case.

12

TESTING THE HYPOTHESIS

12. Testing the Fractal Architecture Hypothesis

12.1 The question

Research question. Do original calculations support, limit or refute the four parts of the Fractal Architecture Hypothesis?

Sections 5 to 11 reported ten calculations alongside the literature. This section gathers them as formal tests. Each was designed either to check whether a proposed mechanism can produce the fractal statistics attributed to it, or to try to break a step of the hypothesis. None uses real-world data directly; all are models whose ingredients are fully known. Their value is that they isolate a logical point: what a mechanism *can* produce, and what a measurement *can* distinguish.

12.2 The ten tests

T1. Measuring dimension. Box counting recovered the dimensions of exact fractals to within about 3 per cent, but a random walk four million steps long measured only 1.735 against a theoretical 2 (Figure 7). *Supports* the measurability of fractal structure, with a caution that finite samples bias estimates.

T2. Coastlines. A divider walk recovered built-in dimensions of 1.02 to 1.45 to within 0.02 (Figure 14). *Supports* Richardson's method and Mandelbrot's interpretation.

T3. River networks. Drainage carved into a fractal landscape gave a Hack exponent of 0.558, within the observed range of 0.5 to 0.6; a random network gave 0.645 (Figure 15). *Supports* H2: a simple physical mechanism reproduces natural statistics. It also shows that exponents are easy to obtain from different models.

T4. Clouds. Thresholding a rough random field gave an area–perimeter dimension of 1.34, matching both theory and Lovejoy's observation (Figure 16). *Supports* H2.

T5. Criticality. A sandpile with no tuned parameters produced power-law avalanches, from single topplings to more than ten thousand, with a cutoff set by system size (Figure 17). *Supports* H2 and illustrates H4.

T6. Universality. A diffusion-limited aggregate and a percolation cluster gave dimensions of 1.733 and 1.882, close to their class values of 1.71 and 1.896 (Figure 21). *Strongly supports* H3 for systems within a universality class.

T7. Mechanism identification. An aggregate and a Cantor dust shared a dimension of 1.64 but differed in lacunarity by about a factor of two (Figure 22). *Limits* H3: equal dimension does not imply equal mechanism; texture measures are needed.

T8. False positives. Random discs, which are not fractal, gave a dimension of 1.5 with R^2 above 0.995 over 1.3 decades (Figure 25). *Limits* H1 as commonly evidenced: many reported fractals would not survive the two-decade test.

T9. Distributions. A lognormal mimicked a power law until a likelihood-ratio test on a large sample separated them, and a true power law could not be distinguished from a flexible lognormal (Figure 26). *Limits* scale-free claims.

T10. Cosmic boundedness. A hierarchical model with built-in dimension 2 gave a measured dimension near 2 inside clusters, rising to 3 at about 8 cluster radii (Figure 19). *Supports* H4 and illustrates how real surveys detect the end of cosmic fractality.

TEST	QUESTION	RESULT FROM THIS REPORT'S MODELS	VERDICT
T1	Can dimension be measured reliably?	Box counting recovers known dimensions to within ~3%; finite samples bias estimates low	Supports, with caution
T2	Do measured coastlines behave as Richardson found?	Divider method recovers built-in dimensions 1.02–1.45 to within 0.02	Supports
T3	Does a simple process yield natural-looking statistics?	Drainage on a fractal landscape gives Hack exponent 0.56; random networks give 0.65	Supports
T4	Can geometry follow from noise statistics alone?	Cloud-like regions give $D = 1.35$, matching both theory and observation	Supports
T5	Do power laws arise without tuning?	A sandpile self-organizes to avalanches spanning four decades	Supports
T6	Do universality classes really share dimensions?	DLA gives $D \approx 1.73$ and percolation $D \approx 1.88$, as theory predicts	Strongly supports
T7	Does dimension identify mechanism?	A grown cluster and a cut-out dust share $D = 1.64$ but differ in lacunarity	Limits the claim
T8	Can non-fractals pass as fractals?	Random discs give ' $D = 1.50$ ' with $R^2 > 0.995$ over 1.3 decades	Limits the claim
T9	Can a lognormal pass as a power law?	Only a large sample and a likelihood test tell them apart	Limits the claim
T10	Is cosmic fractality unbounded?	A hierarchical model shows $D_2 \approx 2$ inside clusters, rising to 3 beyond them	Bounds the claim

Figure 27. Scorecard of the ten tests. Six support the prevalence and mechanism parts of the hypothesis; four limit or bound the claims that fractal similarity reveals shared causes, and that fractality extends to all scales.

Source: H Heuristics calculations; each test is described below with a reference to its figure.

Two further calculations bear on persistence rather than on the four hypotheses directly. Finite branching networks approached the metabolic 3/4 exponent only at 30 or more levels (Figure 12), qualifying the strongest biological claim for fractal networks. And branching hierarchies reached a million points with 261 times less material than direct connections (Figure 24), supporting function as a reason for persistence.

12.3 What the tests show

Table 15. The four parts of the hypothesis against the evidence

Part	Claim	Evidence from literature and tests	Verdict
H1	Fractal structure over ≥ 2 decades occurs in all four domains	Clear in life, Earth and the universe; weaker and often short-ranged in civilization; many published cases fail T8-type scrutiny	Supported, with exceptions (B)
H2	It can be traced to the seven mechanisms	Every well-documented case has at least one mechanism that reproduces it (T2–T6)	Supported (B)
H3	Fractal similarity tracks mechanism	Holds within universality classes (T6) and partly for functional optima; fails for dimension alone (T7) and for descriptive resemblances	Partly supported (B within classes; D across them)
H4	All physical fractals are bounded; no domain is fractal at all scales	Cutoffs found in every system; cosmic homogeneity above ~ 100 Mpc (T10)	Supported (A)

The tests were designed to be capable of failing. T7–T9 show that the methods used to establish fractal similarity can mislead, which is why H3 receives a split verdict.

The pattern of results is consistent. Fractal structure is real, measurable and widely generated by simple mechanisms (H1, H2). It is always bounded (H4). The claim that most matters for the foundational level, that fractal similarity tracks shared mechanism (H3), holds in its strongest form only within universality classes, holds in a weaker form for systems solving the same optimization problem, and fails when the evidence is a shared appearance or a single shared dimension. The hypothesis survives, but it is narrowed: fractal geometry is a reliable guide to shared causes only when used with the full set of tests.

13

HYPOTHESES AND RESEARCH AGENDA



13. Hypotheses and Research Agenda

13.1 The question

Research question. What specific, testable propositions follow from the report's analysis, and what research would decide them?

The report's verdict on the Fractal Architecture Hypothesis is split: supported at the physical and biological level, partly supported for the claim that fractal similarity tracks mechanism, and open at the foundational level. Progress requires propositions that can fail. This section states ten, each with a prediction and a way of testing it. They are the report's own hypotheses and are labeled as such.

13.2 Ten propositions

Table 16. Ten testable propositions arising from the report

#	Proposition	Prediction	How to test it
P1	Mechanism, not domain, is the natural unit of fractal comparison	Systems sharing a mechanism class will share exponents more closely than systems sharing a domain	Meta-analysis of published exponents grouped by mechanism versus by domain
P2	Cutoffs are set by identifiable competing processes	Inner and outer cutoffs will scale predictably with the competing process (e.g. viscosity, diffusion length, system age)	Measure cutoffs across systems in which the competing parameter varies
P3	Biological fractal ratios track costs and risks	Departures from optimal branching ratios will correlate with the cost of failure (e.g. airway safety margins)	Comparative morphometry across species and organs with different failure costs
P4	Metabolic exponents vary with network depth	Within-clade exponents will drift with body size as finite-size network models predict	Fit size-dependent exponents to large metabolic data sets and compare with network models
P5	Lacunarity and multifractal spectra discriminate mechanisms that dimension cannot	Systems with equal dimension but different mechanisms will differ in texture measures	Blind classification of simulated and real structures using texture measures
P6	Most published fractal dimensions with < 1.5 decades of scaling will not replicate	Re-measurement at wider ranges will change many reported dimensions or remove the scaling	Systematic replication of a random sample of published fractal claims
P7	The homogeneity scale reflects the primordial power spectrum	The scale at which D_2 reaches 3 will shift with redshift as structure-formation models predict	Next-generation surveys (e.g. DESI, Euclid) measuring $D_2(r)$ across redshift

#	Proposition	Prediction	How to test it
P8	Urban fractality depends on transport technology	Footprint dimensions and their scaling ranges will differ systematically between car-era and pre-car cities	Historical and cross-national comparison of urban footprints at fixed resolution
P9	Cross-scale resemblances without a physical link belong to the statistical or descriptive kind of unity	Resemblances such as neurons and the cosmic web will fail at least one texture or mechanism test	Apply the five tests to prominent cross-scale analogies
P10	Fractal structure in living systems is maintained by development, not encoded as geometry	Perturbing branching genes will change the number of levels and cutoffs before changing ratios	Developmental genetics in model plants and organs (cf. the Romanesco work)

Each proposition can be falsified by the test listed. P1, P5 and P9 bear most directly on the foundational level of the Fractal Architecture Hypothesis.

13.3 Priorities

Three of the propositions are especially important for deciding the central question. P1 and P5 together would establish whether fractal comparison can be made mechanism-diagnostic. If systems grouped by mechanism show tighter agreement than systems grouped by domain, and if texture measures can identify the mechanism of an unknown structure, fractal geometry becomes a discovery tool rather than a descriptive one. P6 addresses the evidence base itself. A replication program for published fractal claims, analogous to replication efforts in psychology and medicine, would tell the field how much of its catalogue survives the two-decade test. P9 tackles the most seductive analogies directly, and is the proposition most likely to change public understanding.

13.4 Methods the field needs

- **Reporting standards.** Every fractal claim should state the scaling range in decades, the inner and outer cutoffs, the estimation method, an uncertainty interval and the alternatives tested, and should show local slopes against scale.
- **Null models.** Claims should be compared with randomized versions of the same data, such as shuffled networks or random arrangements of the same objects, to show that the fractal signature is not produced by size distributions or sampling alone.
- **Texture measures.** Lacunarity, generalized dimensions, branching ratios and topological measures should accompany dimension whenever systems are compared.
- **Open data and code.** Fractal estimates are sensitive to preprocessing; publishing images, thresholds and code would allow independent re-analysis.
- **Mechanistic models.** A fractal claim gains most when paired with a generative model that reproduces both the scaling and the cutoffs, as optimal channel networks do for rivers.

14

IMPLICATIONS



14. Implications

14.1 The question

Research question. What does the report's analysis mean for science, for design and medicine, and for the Architecture of Creation series?

14.2 For science

The first implication is methodological. Fractal geometry should be used as a measurement discipline, not as a metaphor. Its distinctive contribution is the ability to quantify irregular, multiscale structure with a small number of exponents, and that contribution is lost when 'fractal' becomes a synonym for 'complex-looking'. The ladder of claims and the five tests in Section 4 are intended as a common standard across disciplines.

The second implication concerns interdisciplinary unity. The deepest connections between domains run through mechanisms and their mathematics, not through shapes. Laplacian growth unites electrodeposition, lightning and viscous fingering; transport optimization unites vessels, airways and rivers; cascades unite turbulence, rainfall and perhaps markets; proportional growth unites city sizes, firm sizes and network degrees. A research program organized around these mechanism classes, rather than around fractal appearance, would be more likely to find genuine shared principles.

The third implication is that characteristic scales deserve as much attention as scale invariance. The homogeneity scale of the universe, the Kolmogorov scale of turbulence and the alveolar size of the lung each encode physics that the scaling range does not. Fractal analysis that reports only a dimension discards them.

14.3 For design, engineering and medicine

Fractal principles already shape technology. [Fractal antennas](#) use self-similar shapes to receive several frequency bands in a compact space. Heat exchangers, microfluidic chips and fuel-cell flow fields use branching networks modelled on Murray's law to distribute fluids with minimal pumping power. Porous electrodes and catalysts use fractal surfaces to maximize reactive area. Computer graphics has used fractal terrain and plant models for four decades. In each case, the useful insight is the one identified in Section 10: hierarchical branching and folding achieve large reach or area with little material.

In medicine, fractal measures are being explored as biomarkers: the dimension of retinal vasculature, the multifractality of heart rate, the irregularity of tumour boundaries and the complexity of cortical folding. The evidence varies from promising to established, and the methodological cautions of Section 11 apply in full: clinical fractal measures need standardized resolution, preprocessing and scaling ranges to be

comparable across studies. The finding that healthy physiology is often more variable, not less, than diseased physiology is one of the field's lasting contributions ([Goldberger et al. 2002](#)).

In urban planning, fractal and scaling analyses offer ways to measure sprawl, accessibility and the balance between compact and green space. They are best used comparatively, to track change in one city over time at fixed resolution, rather than to compare absolute dimensions between cities measured differently.

14.4 For the Architecture of Creation series

The series argues that reality produces recurring forms because a constrained set of processes operates within a constrained set of physical laws. Fractals provide one of the sharpest tests of that argument, and the result is a refinement rather than a confirmation. Recurrence is real, but it is recurrence of *processes*, each with its own mathematical signature, rather than recurrence of a single fractal form. The series framework of constraints, generators and filters maps directly onto the report's findings: the seven mechanisms are generators; optimization, selection and stability are filters; and geometric limits, conservation laws and the statistics of random processes are constraints.

Table 17. Implications for the other reports in the series

Report	Fractal structures it discusses	Implication from this report
The Architecture of Creation in the Universe	Cosmic web, galaxy clustering, hierarchical structure formation	Fractal clustering is real over a limited range; homogeneity above ~100 Mpc is established
The Architecture of Life	Vascular, airway and plant networks; metabolic scaling	Transport optimization is the shared principle; the universal 3/4 law should be graded C
The Architecture of Earth	Rivers, coastlines, clouds, turbulence, earthquakes	Earth's fractals are the best measured; their exponents are reproduced by simple mechanisms
The Architecture of Entropy	Cascades, dissipative structures, criticality	Fractal structure often marks systems that dissipate energy efficiently or sit near criticality
The Architecture of Civilization	City sizes, urban scaling, networks, markets	Heavy tails are common, strict fractality is rare; treat scale-free claims with caution
The Mathematical Architecture of Reality	Scaling laws, universality, power-law testing	Fractals exemplify universality as the source of mathematics' cross-domain reach

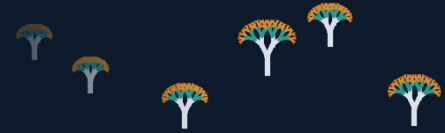
The table suggests where earlier reports' fractal claims can be strengthened, qualified or regraded in light of this report's tests.

14.5 For H Heuristics

For H Heuristics research more broadly, the report recommends four practices. Grade every scaling claim on the ladder of Figure 10. Report scaling ranges and cutoffs alongside exponents. Distinguish universal, functional, statistical and descriptive kinds of unity when comparing systems across domains. And treat striking visual analogies as hypotheses to be tested, never as conclusions.

15

CONCLUSION



15. Conclusion

15.1 The answer to the central question

The report asked whether fractal geometry provides a unifying way to understand how patterns emerge, repeat and organize across the scales of reality. The answer is yes, in a specific and limited sense. Fractal geometry is a unifying *language*: it measures irregular, multiscale structure in coastlines, lungs, rivers, clouds, galaxies and cities with the same small set of tools, and that alone has transformed several sciences. It also reveals a unifying *architecture*, but the architecture is one of processes rather than shapes. A handful of generative mechanisms, branching for transport, recursive growth, aggregation, criticality, cascades, erosion against resistance and proportional growth, recur across life, Earth, the universe and society. Where two systems share a mechanism class, they share measurable scaling properties, sometimes exactly. Where they share only an appearance, the resemblance fails quantitative tests.

The report's evidence map (Figure 28) summarizes the result. The narrower the claim, the stronger the evidence. That ideal fractals have non-integer dimension is proven. That turbulence is multifractal, that diffusion-limited growth shares a dimension across materials, and that the universe becomes homogeneous above about 100 megaparsecs are established. That clouds, rivers, coastlines and vascular trees are fractal over substantial ranges is well supported. That metabolic scaling, city form or most networks are explained by fractal geometry is contested. That reality is fractal at every scale, or that fractal organization is a deep architecture common to all of nature, is speculative.

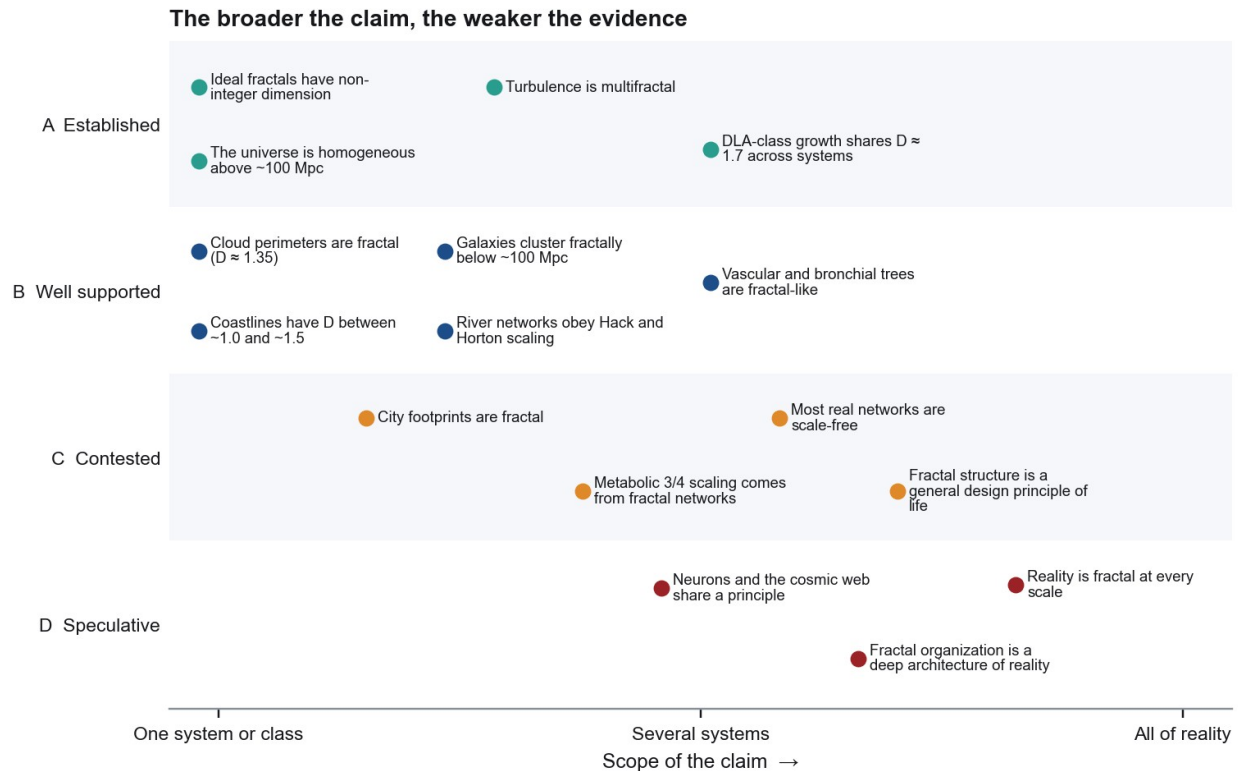


Figure 28. The report's evidence map. Each point is a claim, placed by its evidence grade and by the breadth of its scope. Strong evidence supports narrow claims about specific systems and mechanism classes; the broadest claims about a fractal reality are speculative.

Source: H Heuristics assessment based on Sections 3–12.

15.2 What is established, contested and open

Table 18. What the report establishes, what remains contested and what is open

Established or well supported	Contested	Open
Fractals are exact mathematical objects with well-defined dimensions	That a universal $3/4$ metabolic law follows from fractal networks	Whether fractal organization reflects a general architecture of reality
Many natural systems are fractal or multifractal over two or more decades	That cities have a characteristic fractal form	Whether texture measures can make fractal analysis mechanism-diagnostic
Every physical fractal has cutoffs set by competing physics	That most real networks are scale-free	How far self-organized criticality explains natural power laws
Universality classes produce identical exponents across materials	That fractal dimension can authenticate or date artworks	Whether cross-scale analogies such as brain and cosmos share any principle
Branching hierarchies supply volumes far more cheaply than direct connections	That chromatin is a fractal globule	How the homogeneity scale evolves with cosmic time
The universe is homogeneous above roughly 100 Mpc	That markets are multifractal cascades	How many published fractal claims would replicate

The columns are graded A–B, C and D respectively in the terms of Table 1.

15.3 What the report contributes

The report makes five contributions. It distinguishes three levels of the Fractal Architecture Hypothesis and shows that most confusion arises from moving between them without evidence. It identifies seven generative mechanisms with testable signatures. It proposes a five-rung ladder of fractal claims and five tests for any claim of shared architecture. It distinguishes four kinds of fractal unity: universal, functional, statistical and descriptive. And it provides ten original calculations, six of which support the prevalence and mechanism parts of the hypothesis and four of which show precisely how fractal similarity can mislead.

15.4 How this informs the series

For the Architecture of Creation series, fractals turn out to be an ideal case study of the framework's central claim, and of its limits. Recurring form does reveal recurring process. But the recurrence must be established quantitatively, the processes must be identified, and the boundaries of each pattern must be respected. The most persuasive-looking cases, such as galaxies that resemble neurons, are often the least informative, while the least glamorous, such as a shared exponent in zinc deposits and electrical discharges, are the ones that genuinely unify. The same lesson applies to every cross-domain comparison in the series.

15.5 Closing

Mandelbrot's insight was that the roughness of the world is not a failure of geometry but a geometry of its own. Half a century later, that insight has been both vindicated and disciplined. Vindicated, because fractal structure is everywhere that branching, aggregation, turbulence and growth are at work, and because fractal measures have become standard tools across the sciences. Disciplined, because every real fractal ends, because many claimed fractals are not, and because the same shape can hide very different causes. What remains is a more interesting picture than the slogan of a fractal universe. Reality is not one pattern repeated at every scale. It is a small number of ways of building, repeated across very different materials, each leaving a mathematical signature that can be measured and compared. Fractal geometry is how those signatures are read.

Final verdict. Fractal geometry reveals a genuine but bounded architecture of creation: not one shape repeated through all of reality, but a few generative processes recurring across domains, recognizable by their scaling and limited by the physics that ends every fractal.

References

The list below gives full details for every primary source cited by DOI in the text, generated from the DOI registry metadata and checked against registered titles. Reference articles and works cited by other stable links are linked in the text where they are used and are not repeated here. Books discussed include Mandelbrot's *The Fractal Geometry of Nature* (1982), Richardson's posthumous 'The problem of contiguity' (1961), Batty and Longley's *Fractal Cities* (1994), Eglash's *African Fractals* (1999), Prusinkiewicz and Lindenmayer's *The Algorithmic Beauty of Plants* (1990) and Peebles' *The Large-Scale Structure of the Universe* (1980).

- Albert, R. & Barabási, A. L. (2002). Statistical mechanics of complex networks. *Reviews of Modern Physics* 74, 47-97. [doi:10.1103/RevModPhys.74.47](https://doi.org/10.1103/RevModPhys.74.47)
- Arcaute, E. et al. (2015). Constructing cities, deconstructing scaling laws. *Journal of The Royal Society Interface* 12, 20140745. [doi:10.1098/rsif.2014.0745](https://doi.org/10.1098/rsif.2014.0745)
- Avnir, D. et al. (1998). Is the Geometry of Nature Fractal? *Science* 279, 39-40. [doi:10.1126/science.279.5347.39](https://doi.org/10.1126/science.279.5347.39)
- Azpeitia, E. et al. (2021). Cauliflower fractal forms arise from perturbations of floral gene networks. *Science* 373, 192-197. [doi:10.1126/science.abg5999](https://doi.org/10.1126/science.abg5999)
- Bak, P. et al. (1987). Self-organized criticality: An explanation of the $1/f$ noise. *Physical Review Letters* 59, 381-384. [doi:10.1103/PhysRevLett.59.381](https://doi.org/10.1103/PhysRevLett.59.381)
- Banavar, J. R. et al. (1999). Size and form in efficient transportation networks. *Nature* 399, 130-132. [doi:10.1038/20144](https://doi.org/10.1038/20144)
- Barabási, A. L. & Albert, R. (1999). Emergence of Scaling in Random Networks. *Science* 286, 509-512. [doi:10.1126/science.286.5439.509](https://doi.org/10.1126/science.286.5439.509)
- Barnsley, M. F. & Demko, S. (1985). Iterated function systems and the global construction of fractals. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences* 399, 243-275. [doi:10.1098/rspa.1985.0057](https://doi.org/10.1098/rspa.1985.0057)
- Batty, M. & Longley, P. A. (1987). Fractal-Based Description of Urban Form. *Environment and Planning B: Planning and Design* 14, 123-134. [doi:10.1068/b140123](https://doi.org/10.1068/b140123)
- Batty, M. (2008). The Size, Scale, and Shape of Cities. *Science* 319, 769-771. [doi:10.1126/science.1151419](https://doi.org/10.1126/science.1151419)
- Bettencourt, L. M. A. (2013). The Origins of Scaling in Cities. *Science* 340, 1438-1441. [doi:10.1126/science.1235823](https://doi.org/10.1126/science.1235823)
- Bettencourt, L. M. A. et al. (2007). Growth, innovation, scaling, and the pace of life in cities. *Proceedings of the National Academy of Sciences* 104, 7301-7306. [doi:10.1073/pnas.0610172104](https://doi.org/10.1073/pnas.0610172104)
- Bond, J. R. et al. (1996). How filaments of galaxies are woven into the cosmic web. *Nature* 380, 603-606. [doi:10.1038/380603a0](https://doi.org/10.1038/380603a0)
- Bonnet, E. et al. (2001). Scaling of fracture systems in geological media. *Reviews of Geophysics* 39, 347-383. [doi:10.1029/1999RG000074](https://doi.org/10.1029/1999RG000074)
- Broido, A. D. & Clauset, A. (2019). Scale-free networks are rare. *Nature Communications* 10, 1017. [doi:10.1038/s41467-019-08746-5](https://doi.org/10.1038/s41467-019-08746-5)
- Brown, J. H. et al. (2004). Toward a metabolic theory of ecology. *Ecology* 85, 1771-1789. [doi:10.1890/03-9000](https://doi.org/10.1890/03-9000)
- Cantor, G. (1883). Ueber unendliche, lineare Punktmannichfaltigkeiten. *Mathematische Annalen* 21, 545-591. [doi:10.1007/BF01446819](https://doi.org/10.1007/BF01446819)
- Clauset, A. et al. (2009). Power-Law Distributions in Empirical Data. *SIAM Review* 51, 661-703. [doi:10.1137/070710111](https://doi.org/10.1137/070710111)
- Cristelli, M. et al. (2012). There is More than a Power Law in Zipf. *Scientific Reports* 2, 812. [doi:10.1038/srep00812](https://doi.org/10.1038/srep00812)
- Cuntz, H. et al. (2010). One Rule to Grow Them All: A General Theory of Neuronal Branching and Its Practical Application. *PLoS Computational Biology* 6, e1000877. [doi:10.1371/journal.pcbi.1000877](https://doi.org/10.1371/journal.pcbi.1000877)
- Davis, M. & Peebles, P. J. E. (1983). A survey of galaxy redshifts. V - The two-point position and velocity correlations. *The Astrophysical Journal* 267, 465. [doi:10.1086/160884](https://doi.org/10.1086/160884)
- Dodds, P. et al. (2001). Re-examination of the "3/4 law" of Metabolism. *Journal of Theoretical Biology* 209, 9-27. [doi:10.1006/jtbi.2000.2238](https://doi.org/10.1006/jtbi.2000.2238)
- Dodds, P. S. & Rothman, D. H. (2000). Scaling, Universality, and Geomorphology. *Annual Review of Earth and Planetary Sciences* 28, 571-610. [doi:10.1146/annurev.earth.28.1.571](https://doi.org/10.1146/annurev.earth.28.1.571)

- Edwards, A. M. et al. (2007). Revisiting Lévy flight search patterns of wandering albatrosses, bumblebees and deer. *Nature* 449, 1044-1048. [doi:10.1038/nature06199](https://doi.org/10.1038/nature06199)
- Eloy, C. (2011). Leonardo's Rule, Self-Similarity, and Wind-Induced Stresses in Trees. *Physical Review Letters* 107, 258101. [doi:10.1103/PhysRevLett.107.258101](https://doi.org/10.1103/PhysRevLett.107.258101)
- Faloutsos, M. et al. (1999). On power-law relationships of the Internet topology. *ACM SIGCOMM Computer Communication Review* 29, 251-262. [doi:10.1145/316194.316229](https://doi.org/10.1145/316194.316229)
- Forrest, S. R. & Witten, T. A. (1979). Long-range correlations in smoke-particle aggregates. *Journal of Physics A: Mathematical and General* 12, L109-L117. [doi:10.1088/0305-4470/12/5/008](https://doi.org/10.1088/0305-4470/12/5/008)
- Gabaix, X. (1999). Zipf's Law for Cities: An Explanation. *The Quarterly Journal of Economics* 114, 739-767. [doi:10.1162/003355399556133](https://doi.org/10.1162/003355399556133)
- García-Pérez, G. et al. (2018). Multiscale unfolding of real networks by geometric renormalization. *Nature Physics* 14, 583-589. [doi:10.1038/s41567-018-0072-5](https://doi.org/10.1038/s41567-018-0072-5)
- Gneiting, T. & Schlather, M. (2004). Stochastic Models That Separate Fractal Dimension and the Hurst Effect. *SIAM Review* 46, 269-282. [doi:10.1137/S0036144501394387](https://doi.org/10.1137/S0036144501394387)
- Goldberger, A. L. et al. (2002). Fractal dynamics in physiology: Alterations with disease and aging. *Proceedings of the National Academy of Sciences* 99, 2466-2472. [doi:10.1073/pnas.012579499](https://doi.org/10.1073/pnas.012579499)
- Grassberger, P. & Procaccia, I. (1983). Characterization of Strange Attractors. *Physical Review Letters* 50, 346-349. [doi:10.1103/PhysRevLett.50.346](https://doi.org/10.1103/PhysRevLett.50.346)
- Grassberger, P. & Procaccia, I. (1983). Measuring the strangeness of strange attractors. *Physica D: Nonlinear Phenomena* 9, 189-208. [doi:10.1016/0167-2789\(83\)90298-1](https://doi.org/10.1016/0167-2789(83)90298-1)
- Gutenberg, B. & Richter, C. F. (1944). Frequency of earthquakes in California. *Bulletin of the Seismological Society of America* 34, 185-188*. [doi:10.1785/BSSA0340040185](https://doi.org/10.1785/BSSA0340040185)
- Hack, J. (1957). Studies of longitudinal stream profiles in Virginia and Maryland. *Professional Paper*. [doi:10.3133/pp294B](https://doi.org/10.3133/pp294B)
- Halley, J. M. et al. (2004). Uses and abuses of fractal methodology in ecology. *Ecology Letters* 7, 254-271. [doi:10.1111/j.1461-0248.2004.00568.x](https://doi.org/10.1111/j.1461-0248.2004.00568.x)
- Halsey, T. C. et al. (1986). Fractal measures and their singularities: The characterization of strange sets. *Physical Review A* 33, 1141-1151. [doi:10.1103/PhysRevA.33.1141](https://doi.org/10.1103/PhysRevA.33.1141)
- Hausdorff, F. (1918). Dimension und äußeres Maß. *Mathematische Annalen* 79, 157-179. [doi:10.1007/BF01457179](https://doi.org/10.1007/BF01457179)
- Hentschel, H. & Procaccia, I. (1983). The infinite number of generalized dimensions of fractals and strange attractors. *Physica D: Nonlinear Phenomena* 8, 435-444. [doi:10.1016/0167-2789\(83\)90235-X](https://doi.org/10.1016/0167-2789(83)90235-X)
- Hogg, D. W. et al. (2005). Cosmic Homogeneity Demonstrated with Luminous Red Galaxies. *The Astrophysical Journal* 624, 54-58. [doi:10.1086/429084](https://doi.org/10.1086/429084)
- Holme, P. (2019). Rare and everywhere: Perspectives on scale-free networks. *Nature Communications* 10, 1016. [doi:10.1038/s41467-019-09038-8](https://doi.org/10.1038/s41467-019-09038-8)
- Horton, R. E. (1945). Erosional development of streams and their drainage basins; hydrophysical approach to quantitative morphology. *Geological Society of America Bulletin* 56, 275. [doi:10.1130/0016-7606\(1945\)56\[275:EDOSAT\]2.0.CO;2](https://doi.org/10.1130/0016-7606(1945)56[275:EDOSAT]2.0.CO;2)
- Huo, Y. & Kassab, G. S. (2012). Intraspecific scaling laws of vascular trees. *Journal of The Royal Society Interface* 9, 190-200. [doi:10.1098/rsif.2011.0270](https://doi.org/10.1098/rsif.2011.0270)
- Hurst, H. E. (1951). Long-Term Storage Capacity of Reservoirs. *Transactions of the American Society of Civil Engineers* 116, 770-799. [doi:10.1061/TACEAT.0006518](https://doi.org/10.1061/TACEAT.0006518)
- Hutchinson, J. (1981). Fractals and self similarity. *Indiana University Mathematics Journal* 30, 713. [doi:10.1512/iumj.1981.30.30055](https://doi.org/10.1512/iumj.1981.30.30055)
- Ivanov, P. C. et al. (1999). Multifractality in human heartbeat dynamics. *Nature* 399, 461-465. [doi:10.1038/20924](https://doi.org/10.1038/20924)
- Jones, B. J. T. et al. (2005). Scaling laws in the distribution of galaxies. *Reviews of Modern Physics* 76, 1211-1266. [doi:10.1103/RevModPhys.76.1211](https://doi.org/10.1103/RevModPhys.76.1211)
- Jones-Smith, K. & Mathur, H. (2006). Revisiting Pollock's drip paintings. *Nature* 444, E9-E10. [doi:10.1038/nature05398](https://doi.org/10.1038/nature05398)
- Kantelhardt, J. W. et al. (2002). Multifractal detrended fluctuation analysis of nonstationary time series. *Physica A: Statistical Mechanics and its Applications* 316, 87-114. [doi:10.1016/S0378-4371\(02\)01383-3](https://doi.org/10.1016/S0378-4371(02)01383-3)
- Katz, A. J. & Thompson, A. H. (1985). Fractal Sandstone Pores: Implications for Conductivity and Pore Formation. *Physical Review Letters* 54, 1325-1328. [doi:10.1103/PhysRevLett.54.1325](https://doi.org/10.1103/PhysRevLett.54.1325)
- Kirchner, J. W. (1993). Statistical inevitability of Horton's laws and the apparent randomness of stream channel networks. *Geology* 21, 591. [doi:10.1130/0091-7613\(1993\)021<0591:SIOHSL>2.3.CO;2](https://doi.org/10.1130/0091-7613(1993)021<0591:SIOHSL>2.3.CO;2)
- Kiselev, V. G. et al. (2003). Is the brain cortex a fractal? *NeuroImage* 20, 1765-1774. [doi:10.1016/S1053-8119\(03\)00380-X](https://doi.org/10.1016/S1053-8119(03)00380-X)

- Kleiber, M. (1932). Body size and metabolism. *Hilgardia* 6, 315-353. [doi:10.3733/hilg.v06n11p315](https://doi.org/10.3733/hilg.v06n11p315)
- Kolmogorov, A. N. (1991). The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers (trans.; original 1941). *Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences* 434, 9-13. [doi:10.1098/rspa.1991.0075](https://doi.org/10.1098/rspa.1991.0075)
- Kolokotronis, T. et al. (2010). Curvature in metabolic scaling. *Nature* 464, 753-756. [doi:10.1038/nature08920](https://doi.org/10.1038/nature08920)
- Kondev, J. & Henley, C. L. (1995). Geometrical Exponents of Contour Loops on Random Gaussian Surfaces. *Physical Review Letters* 74, 4580-4583. [doi:10.1103/PhysRevLett.74.4580](https://doi.org/10.1103/PhysRevLett.74.4580)
- Leitão, J. C. et al. (2016). Is this scaling nonlinear? *Royal Society Open Science* 3, 150649. [doi:10.1098/rsos.150649](https://doi.org/10.1098/rsos.150649)
- Libeskind, N. I. et al. (2018). Tracing the cosmic web. *Monthly Notices of the Royal Astronomical Society* 473, 1195-1217. [doi:10.1093/mnras/stx1976](https://doi.org/10.1093/mnras/stx1976)
- Lieberman-Aiden, E. et al. (2009). Comprehensive Mapping of Long-Range Interactions Reveals Folding Principles of the Human Genome. *Science* 326, 289-293. [doi:10.1126/science.1181369](https://doi.org/10.1126/science.1181369)
- Lindenmayer, A. (1968). Mathematical models for cellular interactions in development I. Filaments with one-sided inputs. *Journal of Theoretical Biology* 18, 280-299. [doi:10.1016/0022-5193\(68\)90079-9](https://doi.org/10.1016/0022-5193(68)90079-9)
- Lovejoy, S. (1982). Area-Perimeter Relation for Rain and Cloud Areas. *Science* 216, 185-187. [doi:10.1126/science.216.4542.185](https://doi.org/10.1126/science.216.4542.185)
- Makse, H. A. et al. (1995). Modelling urban growth patterns. *Nature* 377, 608-612. [doi:10.1038/377608a0](https://doi.org/10.1038/377608a0)
- Mandelbrot, B. (1963). The Variation of Certain Speculative Prices. *The Journal of Business* 36, 394. [doi:10.1086/294632](https://doi.org/10.1086/294632)
- Mandelbrot, B. (1967). How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension. *Science* 156, 636-638. [doi:10.1126/science.156.3775.636](https://doi.org/10.1126/science.156.3775.636)
- Mandelbrot, B. B. & Van Ness, J. W. (1968). Fractional Brownian Motions, Fractional Noises and Applications. *SIAM Review* 10, 422-437. [doi:10.1137/1010093](https://doi.org/10.1137/1010093)
- Mandelbrot, B. B. (1974). Intermittent turbulence in self-similar cascades: divergence of high moments and dimension of the carrier. *Journal of Fluid Mechanics* 62, 331-358. [doi:10.1017/S0022112074000711](https://doi.org/10.1017/S0022112074000711)
- Mandelbrot, B. B. (1980). FRACTAL ASPECTS OF THE ITERATION OF $z \rightarrow \Lambda z(1 \pm z)$ FOR COMPLEX Λ AND z . *Annals of the New York Academy of Sciences* 357, 249-259. [doi:10.1111/j.1749-6632.1980.tb29690.x](https://doi.org/10.1111/j.1749-6632.1980.tb29690.x)
- Mark, D. M. & Aronson, P. B. (1984). Scale-dependent fractal dimensions of topographic surfaces: An empirical investigation, with applications in geomorphology and computer mapping. *Journal of the International Association for Mathematical Geology* 16, 671-683. [doi:10.1007/BF01033029](https://doi.org/10.1007/BF01033029)
- Masters, B. R. (2004). Fractal Analysis of the Vascular Tree in the Human Retina. *Annual Review of Biomedical Engineering* 6, 427-452. [doi:10.1146/annurev.bioeng.6.040803.140100](https://doi.org/10.1146/annurev.bioeng.6.040803.140100)
- Matsushita, M. et al. (1984). Fractal Structures of Zinc Metal Leaves Grown by Electrodeposition. *Physical Review Letters* 53, 286-289. [doi:10.1103/PhysRevLett.53.286](https://doi.org/10.1103/PhysRevLett.53.286)
- Mauroy, B. et al. (2004). An optimal bronchial tree may be dangerous. *Nature* 427, 633-636. [doi:10.1038/nature02287](https://doi.org/10.1038/nature02287)
- Meneveau, C. & Sreenivasan, K. R. (1987). Simple multifractal cascade model for fully developed turbulence. *Physical Review Letters* 59, 1424-1427. [doi:10.1103/PhysRevLett.59.1424](https://doi.org/10.1103/PhysRevLett.59.1424)
- Mirny, L. A. (2011). The fractal globule as a model of chromatin architecture in the cell. *Chromosome Research* 19, 37-51. [doi:10.1007/s10577-010-9177-0](https://doi.org/10.1007/s10577-010-9177-0)
- Mota, B. & Herculano-Houzel, S. (2015). Cortical folding scales universally with surface area and thickness, not number of neurons. *Science* 349, 74-77. [doi:10.1126/science.aaa9101](https://doi.org/10.1126/science.aaa9101)
- Murray, C. D. (1926). The Physiological Principle of Minimum Work. *Proceedings of the National Academy of Sciences* 12, 207-214. [doi:10.1073/pnas.12.3.207](https://doi.org/10.1073/pnas.12.3.207)
- Måløy, K. J. et al. (1985). Viscous Fingering Fractals in Porous Media. *Physical Review Letters* 55, 2688-2691. [doi:10.1103/PhysRevLett.55.2688](https://doi.org/10.1103/PhysRevLett.55.2688)
- Niemeyer, L. et al. (1984). Fractal Dimension of Dielectric Breakdown. *Physical Review Letters* 52, 1033-1036. [doi:10.1103/PhysRevLett.52.1033](https://doi.org/10.1103/PhysRevLett.52.1033)
- Ntelis, P. et al. (2017). Exploring cosmic homogeneity with the BOSS DR12 galaxy sample. *Journal of Cosmology and Astroparticle Physics* 2017, 019-019. [doi:10.1088/1475-7516/2017/06/019](https://doi.org/10.1088/1475-7516/2017/06/019)
- Oboukhov, A. M. (1962). Some specific features of atmospheric turbulence. *Journal of Fluid Mechanics* 13, 77-81. [doi:10.1017/S0022112062000506](https://doi.org/10.1017/S0022112062000506)
- Peng, C. K. et al. (1994). Mosaic organization of DNA nucleotides. *Physical Review E* 49, 1685-1689. [doi:10.1103/PhysRevE.49.1685](https://doi.org/10.1103/PhysRevE.49.1685)

- Pfeifer, P. & Avnir, D. (1983). Chemistry in noninteger dimensions between two and three. I. Fractal theory of heterogeneous surfaces. *The Journal of Chemical Physics* 79, 3558-3565. [doi:10.1063/1.446210](https://doi.org/10.1063/1.446210)
- Pietronero, L. (1987). The fractal structure of the universe: Correlations of galaxies and clusters and the average mass density. *Physica A: Statistical Mechanics and its Applications* 144, 257-284. [doi:10.1016/0378-4371\(87\)90191-9](https://doi.org/10.1016/0378-4371(87)90191-9)
- Richardson, L. F. (1926). Atmospheric diffusion shown on a distance-neighbour graph. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character* 110, 709-737. [doi:10.1098/rspa.1926.0043](https://doi.org/10.1098/rspa.1926.0043)
- Rinaldo, A. et al. (1992). Minimum energy and fractal structures of drainage networks. *Water Resources Research* 28, 2183-2195. [doi:10.1029/92WR00801](https://doi.org/10.1029/92WR00801)
- Rinaldo, A. et al. (2014). Evolution and selection of river networks: Statics, dynamics, and complexity. *Proceedings of the National Academy of Sciences* 111, 2417-2424. [doi:10.1073/pnas.1322700111](https://doi.org/10.1073/pnas.1322700111)
- Rodríguez-Iturbe, I. et al. (1992). Energy dissipation, runoff production, and the three-dimensional structure of river basins. *Water Resources Research* 28, 1095-1103. [doi:10.1029/91WR03034](https://doi.org/10.1029/91WR03034)
- Sapoval, B. et al. (2004). Self-Stabilized Fractality of Seacoasts through Damped Erosion. *Physical Review Letters* 93, 098501. [doi:10.1103/PhysRevLett.93.098501](https://doi.org/10.1103/PhysRevLett.93.098501)
- Savage, V. M. et al. (2008). Sizing Up Allometric Scaling Theory. *PLoS Computational Biology* 4, e1000171. [doi:10.1371/journal.pcbi.1000171](https://doi.org/10.1371/journal.pcbi.1000171)
- Scheidegger, A. E. (1967). A stochastic model for drainage patterns into an intramontane trench. *International Association of Scientific Hydrology. Bulletin* 12, 15-20. [doi:10.1080/02626666709493507](https://doi.org/10.1080/02626666709493507)
- Scrimgeour, M. I. et al. (2012). The WiggleZ Dark Energy Survey: the transition to large-scale cosmic homogeneity. *Monthly Notices of the Royal Astronomical Society* 425, 116-134. [doi:10.1111/j.1365-2966.2012.21402.x](https://doi.org/10.1111/j.1365-2966.2012.21402.x)
- Shen, G. (2002). Fractal dimension and fractal growth of urbanized areas. *International Journal of Geographical Information Science* 16, 419-437. [doi:10.1080/13658810210137013](https://doi.org/10.1080/13658810210137013)
- Shreve, R. L. (1966). Statistical Law of Stream Numbers. *The Journal of Geology* 74, 17-37. [doi:10.1086/627137](https://doi.org/10.1086/627137)
- Smith, T. et al. (1996). Fractal methods and results in cellular morphology — dimensions, lacunarity and multifractals. *Journal of Neuroscience Methods* 69, 123-136. [doi:10.1016/S0165-0270\(96\)00080-5](https://doi.org/10.1016/S0165-0270(96)00080-5)
- Song, C. et al. (2005). Self-similarity of complex networks. *Nature* 433, 392-395. [doi:10.1038/nature03248](https://doi.org/10.1038/nature03248)
- Spehar, B. et al. (2003). Universal aesthetic of fractals. *Computers & Graphics* 27, 813-820. [doi:10.1016/S0097-8493\(03\)00154-7](https://doi.org/10.1016/S0097-8493(03)00154-7)
- Springel, V. et al. (2005). Simulations of the formation, evolution and clustering of galaxies and quasars. *Nature* 435, 629-636. [doi:10.1038/nature03597](https://doi.org/10.1038/nature03597)
- Sreenivasan, K. R. (1991). Fractals and Multifractals in Fluid Turbulence. *Annual Review of Fluid Mechanics* 23, 539-604. [doi:10.1146/annurev.fl.23.010191.002543](https://doi.org/10.1146/annurev.fl.23.010191.002543)
- Stanley, H. E. & Meakin, P. (1988). Multifractal phenomena in physics and chemistry. *Nature* 335, 405-409. [doi:10.1038/335405a0](https://doi.org/10.1038/335405a0)
- Stumpf, M. P. H. & Porter, M. A. (2012). Critical Truths About Power Laws. *Science* 335, 665-666. [doi:10.1126/science.1216142](https://doi.org/10.1126/science.1216142)
- Sylos Labini, F. et al. (1998). Scale-invariance of galaxy clustering. *Physics Reports* 293, 61-226. [doi:10.1016/S0370-1573\(97\)00044-6](https://doi.org/10.1016/S0370-1573(97)00044-6)
- Taylor, R. P. et al. (1999). Fractal analysis of Pollock's drip paintings. *Nature* 399, 422-422. [doi:10.1038/20833](https://doi.org/10.1038/20833)
- Taylor, R. P. et al. (2011). Perceptual and Physiological Responses to Jackson Pollock's Fractals. *Frontiers in Human Neuroscience* 5. [doi:10.3389/fnhum.2011.00060](https://doi.org/10.3389/fnhum.2011.00060)
- Turcotte, D. L. (1986). Fractals and fragmentation. *Journal of Geophysical Research: Solid Earth* 91, 1921-1926. [doi:10.1029/JB091iB02p01921](https://doi.org/10.1029/JB091iB02p01921)
- Vazza, F. & Feletti, A. (2020). The Quantitative Comparison Between the Neuronal Network and the Cosmic Web. *Frontiers in Physics* 8, 525731. [doi:10.3389/fphy.2020.525731](https://doi.org/10.3389/fphy.2020.525731)
- Weibel, E. R. & Gomez, D. M. (1962). Architecture of the Human Lung. *Science* 137, 577-585. [doi:10.1126/science.137.3530.577](https://doi.org/10.1126/science.137.3530.577)
- Weibel, E. R. (1991). Fractal geometry: a design principle for living organisms. *American Journal of Physiology-Lung Cellular and Molecular Physiology* 261, L361-L369. [doi:10.1152/ajplung.1991.261.6.L361](https://doi.org/10.1152/ajplung.1991.261.6.L361)
- West, B. J. et al. (1986). Beyond the principle of similitude: renormalization in the bronchial tree. *Journal of Applied Physiology* 60, 1089-1097. [doi:10.1152/jappl.1986.60.3.1089](https://doi.org/10.1152/jappl.1986.60.3.1089)

- West, G. B. et al. (1997). A General Model for the Origin of Allometric Scaling Laws in Biology. *Science* 276, 122-126. [doi:10.1126/science.276.5309.122](https://doi.org/10.1126/science.276.5309.122)
- West, G. B. et al. (1999). A general model for the structure and allometry of plant vascular systems. *Nature* 400, 664-667. [doi:10.1038/23251](https://doi.org/10.1038/23251)
- White, C. R. & Seymour, R. S. (2003). Mammalian basal metabolic rate is proportional to body mass^{2/3}. *Proceedings of the National Academy of Sciences* 100, 4046-4049. [doi:10.1073/pnas.0436428100](https://doi.org/10.1073/pnas.0436428100)
- Witten, T. A. & Sander, L. M. (1981). Diffusion-Limited Aggregation, a Kinetic Critical Phenomenon. *Physical Review Letters* 47, 1400-1403. [doi:10.1103/PhysRevLett.47.1400](https://doi.org/10.1103/PhysRevLett.47.1400)
- Yadav, J. K. et al. (2010). Fractal dimension as a measure of the scale of homogeneity. *Monthly Notices of the Royal Astronomical Society*, no-no. [doi:10.1111/j.1365-2966.2010.16612.x](https://doi.org/10.1111/j.1365-2966.2010.16612.x)